Int. J. Production Economics 132 (2011) 292–295

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

A unified solution approach for the due date assignment problem with tardy jobs

Christos Koulamas

Department of Decision Sciences and Information Systems, Florida International University, Miami, FL 33199, USA

ARTICLE INFO

Article history:
Received 8 April 2010
Accepted 20 April 2011
Available online 29 April 2011

Keywords:
Weighted number of tardy jobs
Due date assignment

ABSTRACT

We analyze a number of due date assignment problems with the weighted number of tardy jobs objective and show that these problems can be solved in $O(n^2)$ time by dynamic programming. We show that the effects of learning or the effects of past-sequence-dependent setup times can be incorporated into the problem formulation at no additional computational cost. We also show that some single-machine due date assignment problems can be extended to an identical parallel machine setting. Finally, we improve the complexity of the solution algorithms for two other due date assignment problems.

1. Introduction

The importance of operational issues in supply chain management has led to the emerging research area of supply chain scheduling (Hall and Potts, 2003) in which the supplier's system is modeled as a standard single-machine for scheduling purposes.

In contrast with the bulk of the traditional scheduling literature, the job due dates quoted by the suppliers to their customers are treated as endogenous decision variables in supply chain scheduling. When a batch of jobs is to be delivered to the same customer, a common due date d is assigned to all jobs in the batch resulting in the CON due date assignment method. Alternatively, if the jobs in a batch are not correlated, then the objective is to assign to each job j the optimal non-correlated due date d_j resulting in the DIF due date assignment method. For more on these due date assignment methods, the interested reader is referred to Shabtay and Steiner (2008).

A different stream of scheduling research is concerned with exercising the option of rejecting certain jobs from the schedule. In that case, management has the right to reject a job that does not fit the current work plan incurring a rejection penalty. The objective is to determine a sequence so that the summation of a scheduling objective and the job rejection penalties are minimized. For more on scheduling problems with job rejection, the interested reader is referred to Engels et al. (2003).

It is of interest to notice that the single-machine due date assignment problem with the weighted number of tardy jobs objective and the single-machine scheduling problem with job rejection and the total job completion time objective exhibit similar mathematical structure.

Specifically, in both problems an optimal solution partitions the jobs into two job subsets A, B so that an optimal sequence comprises the jobs in subset A sequenced according to an index priority rule followed by the jobs in the complementary subset B sequenced either in arbitrary order or according to the same index priority rule. This structure facilitates the solution of either problem in $O(n^2)$ by dynamic programming (DP). This observation, to be further analyzed in the next section, was first made by Engels et al. (2003) in the context of the scheduling problem with job rejection and also by Koulamas (2010) in the context of the due date assignment problem.

The objective of this paper is to show that the DP algorithms of Engels et al. (2003) and Koulamas (2010) can be applied to a number of other due date assignment problems with the weighted number of tardy jobs objective. Specifically, we will show that the effects of learning or the effects of past-sequence-dependent (p-s-d) setup times can be incorporated into the due date assignment problem formulation at no additional computational cost. We will also extend some of these problems to an identical parallel machine setting and improve the complexity of the solution algorithms for two other due date assignment problems, namely, the single-machine and the identical parallel machine problems with total earliness costs. A summary table of all the applications will be presented in Section 5.

The rest of the paper is organized as follows. Section 2 presents the general structure of the dynamic programming solution algorithm. Section 3 analyzes single-machine due date assignment problems with past-sequence-dependent setup times (Section 3.1), learning effects (section 3.2) and earliness costs (Section 3.3). Section 4 analyzes due date assignment problems in

E-mail address: koulamas@fiu.edu

0925-5273/$ - see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.ijpe.2011.04.023

an identical parallel machine setting and the conclusions of this research are summarized in Section 5.

2. Preliminary results

Consider a scheduling problem with an optimal solution such that the optimal schedule comprises the jobs in a subset A sequenced according to an index priority rule followed by the jobs in the complementary subset B sequenced either in arbitrary order or according to the same index priority rule. The objective value function of this problem can be written as

$$TC(l^*)=\sum_{j=1}^{l^*}\zeta_{[j]}p_{[j]}+\sum_{j=l^*+1}^{n}w_{[j]} \quad (1)$$

where l^* is the optimal cardinality of subset A, $[j]$ denotes the job in the jth position in the sequence, $p_{[j]}$ is the processing time of job $[j]$, $\zeta_{[j]}$ is the position-dependent but job-independent contribution of job $[j]$ to the objective function (its positional weight) for $j \le l^*$ and $w_{[j]}$ is the position-independent contribution of job $[j]$ to the objective function for $j > l^*$.

The structure of the objective function (1) facilitates the solution of the problem in $O(n^2)$ by dynamic programming (DP). This observation was first made by Engels et al. (2003) in the context of a single-machine scheduling problem with the job rejection and the total job completion time objective, to be called the $1/REJ/\sum_{j=1}^{n}C_j$ problem from now on. Actually, Engels et al. (2003) developed their DP algorithm with the more general total weighted completion time objective and an $O(n\sum w_j)$ running time.

In the $1/REJ/\sum_{j=1}^{n}C_j$ problem, any job j can be rejected from the schedule for an additional penalty w_j. Let A and R denote the subsets of the accepted and rejected jobs, respectively, in an optimal solution S^*. Clearly, there is no inserted idle time in S^* and the jobs in A are sequenced in the shortest-processing-time (SPT) order followed by the jobs in R sequenced in any order; assume that $|A|=l^*$ in S^*. The objective function can be written as

$$TC(l^*)=\sum_{j=1}^{l^*}C_{[j]}+\sum_{j=l^*+1}^{n}w_{[j]}=\sum_{j=1}^{l^*}(l^*-j+1)p_{[j]}+\sum_{j=l^*+1}^{n}w_{[j]} \quad (2)$$

where C_j denotes the completion time of job j. It is clear that the objective function (2) exhibits the structure of the objective function (1) with $\zeta_{[j]}=l^*-j+1$.

In the case of the weighted number of tardy jobs objective and the DIF due date assignment method, the problem is denoted as the $1/DIF/\sum_{j=1}^{n}w_jU_j$ problem and the objective is to determine the due date d_j of each job j and a job sequence so that the function

$$TC=b\sum_{j=1}^{n}d_j+\sum_{j=1}^{n}w_jU_j \quad (3)$$

is minimized where w_j is the positive weight of job j, b is the positive unit due date assignment cost and U_j is the tardiness indicator of job j defined as follows: $U_j=1$ if $C_j>d_j$ and $U_j=0$ if $C_j \le d_j$.

Let E and T denote the sets of the non-tardy (early) jobs and tardy jobs, respectively, in an optimal solution S^*. According to Shabtay and Steiner (2006), there is no inserted idle time in S^* and the jobs in E are sequenced in the SPT order followed by the jobs in T sequenced in any order. Furthermore, $d_{[j]}=C_{[j]}$ if $j\in E$ and $d_{[j]}=0$ if $j\in T$. Assume that $|E|=l^*$ in S^*; then, the above properties facilitate the writing of (3) for the $1/DIF/\sum_{j=1}^{n}w_jU_j$ problem as

$$TC(l^*)=b\sum_{j=1}^{l^*}C_{[j]}+\sum_{j=l^*+1}^{n}w_{[j]}=b\sum_{j=1}^{l^*}(l^*-j+1)p_{[j]}+\sum_{j=l^*+1}^{n}w_{[j]} \quad (4)$$

It is clear that the objective function (4) exhibits the structure of the objective function (1) with $\zeta_{[j]}=b(l^*-j+1)$.

Koulamas (2010) solved the $1/DIF/\sum_{j=1}^{n}w_jU_j$ problem in $O(n^2)$ time by implementing a DP algorithm similar to the one by Engels et al. (2003). The $O(n^2)$ DP algorithms of Engels et al. (2003) and Koulamas (2010) can be described as follows.

Assume that the jobs have been re-indexed in the SPT order. Let state (j,k) define a partial schedule on the jobs $j,\dots,n$ subject to the condition that the first job starts at time 0, the jobs are processed with no-inserted idle time and k jobs among them belong to the $A(E)$ set in the $1/REJ/\sum_{j=1}^{n}C_j$ problem (the $1/DIF/\sum_{j=1}^{n}w_jU_j$ problem), where $k\le n-j+1$. Let $F_j(k)$ denote the optimal solution value of any schedule in state (j,k). The initialization is

$$F_j(k)=\begin{cases}0, & \text{if } j=0 \text{ and } k=0,\\ \infty, & \text{otherwise,}\end{cases}$$

and the recursion for $j=n,\dots,1$ and $k=0,\dots,j$ is

$$F_j(k)=\min\{F_{j+1}(k-1)+\delta_{[k]j},F_{j+1}(k)+w_j\} \quad (5)$$

where $\delta_{[k]j}=bkp_j$ for the $1/DIF/\sum_{j=1}^{n}w_jU_j$ problem and $\delta_{[k]j}=kp_j$ for the $1/REJ/\sum_{j=1}^{n}C_j$ problem. The optimal solution is found as $F^*=\min_{0\le k\le n}F_1(k)$. The optimal job subsets and the resulting optimal due dates (when applicable) are found by backtracking.

In the subsequent sections, the above algorithm is applied to a number of due date assignment problems.

3. Single-machine due date assignment problems

3.1. The due date assignment problem with p-s-d setup times

The past-sequence-dependent (p-s-d) setup times $s_{[j]}$ were introduced by Koulamas and Kyparisis (2008) as $s_{[j]}=\varphi\sum_{i=1}^{j-1}p_{[i]}$, $j=2,\dots,n$, $s_{[1]}=0$, where $s_{[j]}$ is the setup time of job $[j]$ and $\varphi\ge 0$ is a normalizing constant. Koulamas and Kyparisis (2008) showed that the SPT sequence is optimal for the makespan $C_{\max}=\max_{j=1,\dots,n}\{C_j\}$ and the $\sum_{j=1}^{n}C_j$ objectives when $s_{[j]}$ $j=1,\dots,n$ are in place, which is in agreement with the optimal SPT ordering of the early jobs in the $1/DIF/\sum_{j=1}^{n}w_jU_j$ problem. The introduction of p-s-d setup times yields the $1/psd,DIF/\sum_{j=1}^{n}w_jU_j$ problem.

It is clear that the job in the jth position in the sequence contributes $\varphi p_{[j]}$ to the actual processing requirement of each one of the subsequent jobs in the sequence. Therefore, if job j is an early job followed by an additional $k-1$ subsequent early jobs, then its contribution to the objective function (4) can be expressed as $kp_j+(k(k-1)/2)\varphi p_j=k[1+\varphi((k-1)/2)]p_j$. Consequently, the recursive Eq. (5) can be applied with $\delta_{[k]j}=bk[1+\varphi((k-1)/2)]p_j$. The above analysis also applies to the $1/CON/\sum_{j=1}^{n}w_jU_j$ problem with a common assignable due date d.

De et al. (1991) and Kahlbacher and Cheng (1993) observed that

$$TC(l^*)=bnC_{[l^*]}+\sum_{j=l^*+1}^{n}w_{[j]}=bn\sum_{j=1}^{l^*}p_{[j]}+\sum_{j=l^*+1}^{n}w_{[j]}. \quad (6)$$

for the $1/CON/\sum_{j=1}^{n}w_jU_j$ problem. The objective function (6) exhibits similar structure with the objective function (3) and is amenable to the introduction of p-s-d setup times resulting in the $1/psd,CON/\sum_{j=1}^{n}w_jU_j$ problem. In that case, the recursive Eq. (5) can be applied with $\delta_{[k]j}=bn[1+\varphi(k-1)]p_j$.

3.2. The due date assignment problem with learning effects

Biskup (1999) was the first to introduce learning to scheduling problems. Biskup (1999) incorporated the learning phenomenon by defining the actual processing time of job j when scheduled in position r in the sequence, $p_{j[r]}$, as $p_{j[r]}=p_j r^{\alpha}$ where $\alpha<0$ is the applicable learning rate. We denote the presence of learning by inserting the term LE in the problem definition. The literature on due date assignment problems with learning effects is surveyed in the review papers of Gordon et al. (2002) and Biskup (2008). The bulk of this literature deals with the total earliness/tardiness objective. It is well known that, in the presence of learning, the makespan $C_{\max}$ is no longer a constant and that the SPT sequence is optimal for the $1/LE/C_{\max}$ and the $1/LE/\sum_{j=1}^{n} C_j$ problems (Mosheiov (2001) and Biskup (1999), respectively). The objective function (6) can be written as

$$TC(l^*)=bnC_{[l*]}+\sum_{j=l^*+1}^{n} w_{[j]}=bn\sum_{j=1}^{l^*} p_{[j]}j^{\alpha}+\sum_{j=l^*+1}^{n} w_{[j]}. \tag{7}$$

in the case of the $1/LE,CON/\sum_{j=1}^{n} w_jU_j$ problem. The structure of the objective function (7) facilitates the solution of the $1/LE,CON/\sum_{j=1}^{n} w_jU_j$ problem in $O(n^2)$ time by utilizing a forward DP algorithm similar to the backward DP algorithms of Engels et al. (2003) and Koulamas (2010) in which the recursion for $j=1,\ldots,n$ and $k=0,\ldots,j$ is

$$F_j(k)=\min\{F_{j-1}(k-1)+nbk^{\alpha}p_j, F_{j-1}(k)+w_j\} \tag{8}$$

and the optimal solution is $F^*=\min_{0\le k\le n} F_n(k)$.

3.3. The due date assignment problem with earliness costs

A variant of the $1/CON/\sum_{j=1}^{n} w_jU_j$ problem with total earliness costs has been analyzed and solved in $O(n^4)$ time by Kahlbacher and Cheng (1993). In that case, the objective function (6) generalizes to

$$TC(l^*)=bnC_{[l^*]}+\sum_{j=l^*+1}^{n} w_{[j]}+\sum_{j=1}^{n} E_j=bn\sum_{j=1}^{l^*} p_{[j]}+\sum_{j=l^*+1}^{n} w_{[j]}+\sum_{j=1}^{l^*}(j-1)p_{[j]}, \tag{9}$$

where $E_j=\max(0,d-C_j)$ denotes the earliness of job j. The resulting problem is denoted as the $1/CON/\sum_{j=1}^{n} E_j+\sum_{j=1}^{n} w_jU_j$ problem. It is evident from Eq. (9) that if job j is an early job preceded by an additional $k-1$ early jobs, then, its contribution to the objective function (9) can be expressed as $(k-1+nb)p_j$. It is also well known that the total earliness cost is minimized when the jobs are sequenced in the Longest Processing Time (LPT) order. The last early job is actually a just-in-time job. For more about due date assignment with just-in-time scheduling the reader is referred to Tuong and Soukhal (2010) and the references therein.

A forward DP algorithm can be used to solve the $1/CON/\sum_{j=1}^{n} E_j+\sum_{j=1}^{n} w_jU_j$ problem in $O(n^2)$ time by first indexing the jobs according to the LPT order and then replacing the recursive Eq. (8) with the recursive equation $F_j(k)=\min\{F_{j-1}(k-1)+(k-1+nb)p_j, F_{j-1}(k)+w_j\}$.

4. Due date assignment problems with parallel machines

The $1/DIF/\sum_{j=1}^{n} w_jU_j$ problem can be extended to an identical parallel machine (Pm) setting. It is well known that the $Pm//\sum_{j=1}^{n} C_j$ problem is solved by implementing the generalized SPT (GSPT) rule according to which the jobs are first sorted in SPT order and then assigned to the machines using list scheduling (Conway, et al. 1967). This observation facilitates the writing of the objective function for the $Pm/DIF/\sum_{j=1}^{n} w_jU_j$ problem as

$$TC(l^*)=b\sum_{j=1}^{l^*}\left\lceil\frac{l^*-j+1}{m}\right\rceil p_{[j]}+\sum_{j=l^*+1}^{n} w_{[j]}$$

where the $\lceil\ \rceil$ function returns the smallest integer greater than or equal to its argument.

Consequently, the recursive Eq. (5) can be applied with $\delta_{[k]j}=b\lceil (k/m)\rceil p_j$.

We now turn our attention to the $Pm/CON/\sum_{j=1}^{n} E_j+\sum_{j=1}^{n} w_jU_j$ problem without any due date assignment costs solved by Kahlbacher and Cheng (1993) in $O(n^4)$ time. Its objective function can be written as

$$TC=\sum_{j=1}^{n}\max(0,d-C_j)+\sum_{j=1}^{n} w_jU_j. \tag{10}$$

According to Kahlbacher and Cheng (1993), an optimal schedule for the problem comprises either $\lfloor (l^*/m)\rfloor$ or $\lfloor (l^*/m)\rfloor+1$ early jobs on each of the m machines sequenced in the LPT order, followed by the tardy jobs sequenced in any order; the $\lfloor\ \rfloor$ function returns the greatest integer less or equal than its argument. There is inserted idle time before the first early job on each machine (except one) so that the completion time of the last early job on each machine coincides with the common due date d. Consequently, the m shortest early jobs are sequenced immediately prior to the common due date d, the next m shortest early jobs are sequenced immediately prior to the m shortest early jobs, and so on. These observations facilitate the rewriting of (10) as $TC(l^*)=\sum_{j=1}^{l^*}\lfloor ((j-1)/m)\rfloor p_{[j]}+\sum_{j=l^*+1}^{n} w_{[j]}$, which in turn enables the implementation of the recursive Eq. (8) with $\delta_{[k]j}=\lfloor (k-1/m)\rfloor p_j$. It should be pointed out that the term $\lfloor (k-1/m)\rfloor$ becomes negative when evaluated with $k=0$; however, this does not pose a problem because when $k=0$, $F_j(k-1)=\infty$ due to the initialization and therefore the negative $\lfloor (k-1/m)\rfloor$ value is not actually used in the recursion.

5. Conclusions

We analyzed a number of due date assignment problems with the weighted number of tardy jobs objective. We showed that all these problems can be solved in $O(n^2)$ time using either the backward DP algorithms of Engels et al. (2003) and Koulamas (2010) or a similar forward DP algorithm. We showed that the effects of learning or the effects of past-sequence-dependent setup times can be incorporated into the problem formulation at no additional computational cost. We also showed that the $1/DIF/\sum_{j=1}^{n} w_jU_j$ problem can be extended to an identical

Table 1
A summary of the results of this paper.

Problem	DP algorithm	Section in the paper
$1/psd,DIF/\sum_{j=1}^{n} w_jU_j$	Backward	3.1
$1/psd,CON/\sum_{j=1}^{n} w_jU_j$	Backward	3.1
$1/LE,CON/\sum_{j=1}^{n} w_jU_j$	Forward	3.2
$1/CON/\sum_{j=1}^{n} E_j+\sum_{j=1}^{n} w_jU_j$	Forward	3.3
$Pm/DIF/\sum_{j=1}^{n} w_jU_j$	Backward	4
$Pm/CON/\sum_{j=1}^{n} E_j+\sum_{j=1}^{n} w_jU_j$	Forward	4

parallel machine setting. Finally, we improved the complexity of the solution algorithms for the $1/CON/\sum_{j=1}^{n} E_j + \sum_{j=1}^{n} w_j U_j$ and the $Pm/CON/\sum_{j=1}^{n} E_j + \sum_{j=1}^{n} w_j U_j$ problems. Our findings are summarized in Table 1.

Acknowledgments

We would like to thank the referees for their constructive criticism that helped us improve an earlier version of the paper and also for bringing to our attention the references by Engels et al. (2003) and Tuong and Soukhal (2010).

References

Biskup, D., 1999. Single-machine scheduling with learning considerations. European Journal of Operational Research 115, 173–178.
Biskup, D., 2008. A state-of-the-art review of scheduling with learning effects. European Journal of Operational Research 188, 315–329.
Conway, R.W., Maxwell, W.L., Miller, L.W., 1967. Theory of Scheduling. Addison-Wesley, Reading, MA.
De, P., Ghosh, B., Wells, C.E., 1991. Optimal delivery time quotation and order sequencing. Decision Sciences 22, 379–390.
Engels, D.W., Karger, D.R., Kolliopoulos, S.G., Sengupta, S., Uma, R.N., Wein, J., 2003. Techniques for scheduling with rejection. Journal of Algorithms 49, 175–191.
Gordon, V., Proth, J.-M., Chu, C., 2002. A survey of the state-of-the-art of common due date assignment and scheduling research. European Journal of Operational Research 139, 1–25.
Hall, N., Potts, C.N., 2003. Supply chain scheduling: batching and delivery. Operations Research 51, 566–584.
Kahlbacher, H.G., Cheng, T.C.E., 1993. Parallel machine scheduling to minimize costs for earliness and number of tardy jobs. Discrete Applied Mathematics 47, 139–164.
Koulamas, C., 2010. A faster algorithm for a due date assignment problem with tardy jobs. Operations Research Letters 38, 127–128.
Koulamas, C., Kyparisis, G.J., 2008. Single-machine scheduling problems with past-sequence-dependent setup times. European Journal of Operational Research 187, 1045–1049.
Mosheiov, G., 2001. Scheduling problems with a learning effect. European Journal of Operational Research 132, 687–693.
Shabtay, D., Steiner, G., 2006. Two due date assignment problems in scheduling a single machine. Operations Research Letters 34, 683–691.
Shabtay, D., Steiner, G., 2008. The single-machine earliness-tardiness scheduling problem with due date assignment and resource-dependent processing times. Annals of Operations Research 159, 25–40.
Tuong, N.H., Soukhal, A., 2010. Due dates assignment and JIT scheduling with equal-size jobs. European Journal of Operational Research 205, 280–289.