# SEQUENCING DELIVERIES TO MINIMIZE INVENTORY HOLDING COST WITH DOMINANT UPSTREAM SUPPLY CHAIN PARTNER

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#### **Abstract**

This paper studies a two stage supply chain with a dominant upstream partner. Manufacturer is the dominant partner and operates in a Just-in-Time environment. Production is done in a single manufacturing line capable of producing two products without stopping the production for switching from one product to the other. The manufacturer imposes constraints on the distributor by adhering to his favorable production schedule which minimizes his manufacturing cost. Distributor on the other hand caters to retailers' orders without incurring any shortages and is responsible for managing the inventory of finished goods. Adhering to manufacturer's schedule may lead to high inventory carrying costs for the distributor. Distributor's problem, which is to find an optimal distribution sequence which minimizes the distributor's inventory cost under the constraint imposed by the manufacturer is proved NP-Hard by Manoj et al. (2008). Therefore, solving large size problems require efficient heuristics. We develop algorithms for the distribution problem by exploiting its structural properties. We propose two heuristics and use their solutions in the initial population of a genetic algorithm to arrive at solutions with an average deviation of less than 3.5% from the optimal solution for practical size problems.

**Keywords:** Supply chain scheduling, production and distribution system, logistics, genetic algorithm, optimized cross over

# 1. Introduction

There is an increasing emphasis for improving coordination and cooperation among supply chain partners in the supply chain research literature. See, for example, Sarmiento & Nagi (1999), Blumenfeld et al. (1991), Chandra & Fisher (1994), Lee & Chen (2001),

Chang & Lee (2003), Hall & Potts (2003), Agnetis et al. (2006), Chen & Hall (2007), Chen & Vairaktarakis (2005), Li & Xiao (2004), and Dawande et al. (2006).

However, the fact that one of the partners generally assumes a dominant role cannot be ignored. Plewes (2004) states that "The most

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successful supply chain initiatives have tended to be driven by the largest player in the value chain." In the same article, according to Nikos Drakos, research director at market analyst house Gartner, "The relationship is often uneven and there is usually one dominant partner and they will insist that processes are done a specific way. Their partners have to accept the terms of engagement if they want to work with them". "A dominant supply chain leader may use its position of power to force change to occur across the supply chain," observes Defee (2007). Wal-Mart and Dell are examples of dominant partners in their respective supply chains. In such situations, the dominated members of the supply chain will optimize their objectives under the constraints imposed by the dominant members even though individual optimization may not be efficient for the supply chain as a whole. It may be possible for the members in the supply chain to coordinate by means of contracts (Cachon 2003). However, this requires the non-dominant member to find how worse this solution under this scenario is as compared to the coordinated solution.

There are several research studies that focus on study of supply chains in which one partner dominates in an operational context. For example, Manoj et al. (2008) study production and distribution problems in a two-stage supply chain; and compare costs where one of the partners is a dominant partner with the costs of coordinated decisions. Lau et al. (2007a) analyze the pricing decisions for a supply chain in which the manufacturer is the dominant partner; whereas Lau et al. (2007b) focus on pricing decisions for the case of dominant retailer. Xia & Gilbert (2007) study the leadership influence

of manufacturer or dealer on channel structure. Taylor (2006) study the sale timing in a supply chain in a setting where manufacturer is the dominant partner relative to the retailer and have all the bargaining power. Dong et al. (2007) study the reduction of order cost for delivery of goods from manufacturer to the retailer under retailer price leadership and manufacturer price leadership. All of these studies, although focus on different objectives, point to the importance of conducting research that analyze supply chains with a dominant partner.

In this paper we focus on a scenario in which the manufacturer is the dominant partner and the distributor plays the subordinated role. The distributor has to find a distribution schedule based on the production schedule specified by the manufacturer. Specifically, we consider the problem of a distributor who receives products from a manufacturer and then distributes them to the retailers periodically. The manufacturer operates in a Just-in-Time (JIT) environment and is the dominant member of the supply chain.

The manufacturer produces the two products interchangeably in a single manufacturing line and establishes a production schedule that specifies the rate of production of the two products in each time period during the planning horizon. The production schedule established by the manufacturer has to be followed by the distributor for developing a distribution schedule. The method of establishing the production schedule by the manufacturer does not impact the problem studied in this paper. However, manufacturer will establish a production schedule that will minimize his cost of production. Examples of single manufacturing line capable of producing multiple products

include Toyota's Global Body Line (Gardner 1997) and Nissan Integrated Manufacturing systems (NIMS) (nissannews.com).

This paper is organized as follows. Section 2 describes the problem using mathematical notations. It also provides notations for demand from retailers, supplies from the manufacturer, inventory levels and inventory holding costs. The objective of the problem is to minimize total inventory holding costs. The objective function and the constraints of the problem are derived in Section 3. The structural properties of the problem to develop heuristics are studied in Section 4 and the heuristics to solve the problem are developed in Section 5. Section 6 discusses the computational experiments. An extension to model is proposed in Section 7 and concluding remarks are presented in Section 8.

#### 2. Problem Scenario

The supply chain we study has n retailers  $R_k$ ; (k = 1, 2, ..., n), they demand two product types,  $P_i$ , (i = 1, 2) periodically. The manufacturer incurs cost in changing production rate and therefore, his ideal production schedule is the one which has minimum number of rate changes. This cost is associated with allocation/ reallocation of work force, coordination with suppliers to ensure timely delivery of raw materials etc. Toyota, for example, produces Camry cars and Sienna minivans on the same production line at Georgetown, Kentucky. Sienna requires more sophisticated assembly operations (Gardner 1997) so a work force exclusive for Sienna is dedicated when it is being manufactured. Frequent changes in rate schedule is not desirable for the manufacturer as it may result in the under utilization of expert

work force while a product that requires less expertise is being manufactured. Note that there is no time lost for setup changes, as the production line is capable of manufacturing multiple products. Hence, there is no setup cost. However, there is a cost to change production rate as mentioned above. Therefore, the manufacturer's ideal production rate is fixed for the entire problem horizon given by  $p_j$ , j=1,2 (equation (2)), which is equal to the average demand.

The planning period, called the distribution cycle in this paper, consists of n time periods of equal lengths t, as shown in Figure 1. All retailers place orders at the beginning of the distribution cycle. The manufacturer establishes a production rate schedule, based on the total demand, that specifies the production rates (quantities) of the two products to be produced in each time period of the distribution cycle. Each retailer is supplied with one truck load of  $P_1$  and  $P_2$  by the distributor only once in a selected time period s, (s = 1, 2, ..., n) in the distribution cycle. The retailer receives the exact number of units of  $P_1$  and  $P_2$  that he has ordered. The products  $P_1$  and  $P_2$  assumed to have approximately the same volume requiring the same space in the truck. Under the constraint imposed by the manufacturer, the distributor has to find an optimal schedule, which will reduce his inventory holding cost, i.e., his objective is find distribution sequence, to  $v^* = (v(1), v(2), ..., v(s), ..., v(n))$ , that minimizes the inventory holding cost, where v(s) is the retailer who is served in time period s.

The assumption that each retailer's total demand for  $P_1$  and  $P_2$  is limited to one truck load (C units) is not too restrictive; it can be relaxed.

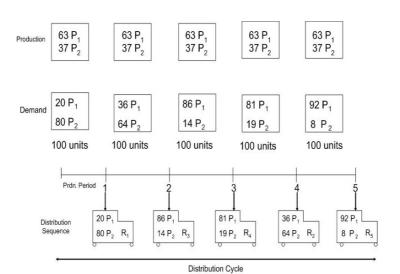


Figure 1 Supply chain network, n=5, C=100

If a retailer has a demand which is more than one truck load then that retailer can be considered as equivalent to multiple retailers. For example if the retailer demand (for  $P_1$  and  $P_2$ ) is 2C units, then it can be assumed that there are two retailers with total demand equal to C units each. However, we develop the model under the assumption that the demand of retailers must be multiple of truck loads. This is a reasonable assumption in automobile industry since the truck capacity, C is small (typically C=10 units of autos). The retailers can be expected to approximate their demand in multiple of C. Note, that the products are manufactured and delivered just-in-time. The products must be transported from the manufacturer's premises (to the nearby distributor's depot) as soon as they completed. Then the products are bundled according to the retailer's demand transported to them by trucks. The main objective in the JIT environment is to perform this distribution operation with minimum

inventory cost.

JIT auto manufacturers in USA typically transport autos in big trucks having the capacity of carrying 10 autos at a time. Assume that a retailer's demand ratio of the two products (say Toyota Camry and Sienna) is 7:3, that is, every 10 autos sold at his location he sells 7 units of Camry and 3 units of Sienna. Suppose his demand in a distribution cycle (typically a month) is 5 truck loads (35 Camry and 15 Sienna). Then the distributor sends 5 truck loads to this retailer each truck carrying 7 Camry and 3 Sienna. If the ratio of demand at a retailer does not sum up to the truck capacity (C = 10), then the composition of auto types can be adjusted between different truck loads delivered during the distribution cycle to closely match the retailer's demand ratio. Consider the demand ratio of the two products for a retailer is 3:1 with the total demand for products  $P_1$  and  $P_2$  are 15 and 5 units, respectively in a distribution cycle, i.e., the total demand of two products is equal to two truck loads. We may consider this particular retailer is equivalent to two retailers each having one truck load. The details of two retailers are given as follows: the demand of products  $P_1$  and  $P_2$  are 7 and 3 units, respectively, for the first retailer whereas for the second retailer the demand for products  $P_1$  and  $P_2$  are 8 and 2 units, respectively.

Our problem is closely related to periodic product distribution which has been a problem of significant industrial relevance that has received intense attention over the years by both academics and practitioners alike. In periodic distribution, a set of products are distributed to retailers on a periodic basis such as once a week, twice a week, and so on. How often should a retailer be visited by the distributor for delivering products during a planning horizon depends upon the retailers' demand, available storage space for products, and preference for days on which to receive the products. An excellent example of periodic product distribution is discount department stores that carry regular products and goods, clothing, and a limited amount of (usually) non-perishable groceries. Others in the periodic distribution sector include those specifically engaged in grocery, food and beverage, automobile and petrochemical industries. The challenge of constructing delivery routes for each day of the planning horizon in periodic distribution is known as Periodic Vehicle Routing Problem (PVRP). In a typical setting, given a set of allowable delivery patterns specified by the retailers, the challenge for the distributor is to select the most suitable delivery pattern for each retailer and deliver the products on the days corresponding to the assigned pattern. In this

study we do not consider routing since each route consists of only one retailer requiring one truck load of products. However, we consider the inventory holding cost at distributor as it is very significant cost for any industry. This scenario is well suited for automobile distribution as the manufacturer (e.g., Toyota) is often a dominant member of the supply chain.

Since the research in periodic product distribution is relevant to our study we review briefly the literature in this domain. A significant number of research pursuits in the classical PVRP have appeared in the literature (Fisher & Jaikumar 1981, Christofides & Beasley 1984, Russell & Gribin 1991, Gaudioso & Paletta 1992, Chao et al. 1995, Cordeau et al. 1997 and Vianna et al. 1999). The classical PVRP focuses on delivering products from a single depot to a number of retailers according to the assigned pattern at a minimum cost with the assumption that the delivery frequencies and quantities are known and fixed. Retailers are assumed to be able to receive the product at any time, meaning that no delivery time windows are required. Every vehicle is allowed to make only one delivery trip per day.

The problem of minimizing the number of delivery routes of a multi-depot PVRP was investigated by Yang & Chu (2000). They assume that the retailers can specify the delivery frequencies in a planning horizon but not the delivery days. The problem is solved by first constructing the routes for each day of the planning horizon and then attempting to reduce the number of routes by combining routes on different days into a single delivery. It should be emphasized that a distributor who owns many depots in a region would typically define an

effective territory for each depot and let each operate independently from the others (Chung & Norback 1991, Hadjiconstantinou & Baldacci 1998). Realistically, however, the product supplies at the distributor's depots can be limited, thus requiring backordering be allowed. A time window can also be specified by retailers for delivery of products by the distributor. Carter et al. (1996) investigated an integrated product distribution problem involving multi-products, allowing for backordering and time-window deliveries in the presence of a single depot and single delivery trip per vehicle on a day. Parthanadee & Logendran (2006) recognized the need for investigating a similar integrated, multi-product, distribution problem, allowing for backordering and time-window deliveries but in the presence of multi-depots and multipledelivery trips per vehicle on a day.

As noted, the focus of this research is to investigate the periodic product distribution in a supply chain setting. The problem is motivated by a third-party distributor with a depot in a region (near manufacturing facility), entrusted with the responsibility of delivering products to a set of retailers, also located in the same region, and sequencing to make such deliveries to retailers on a periodic basis. Supply chains are susceptible to power plays: If the manufacturer dominates, they may dictate terms to the distributor to improve their own performance. The proposed research addresses this question comprehensively by way of developing models and solution techniques for the problem when the manufacturer dominates.

#### 2.1 Demand from Retailers

The demand for the product  $P_i$  for the

planning horizon is given by  $D_j = (d_{j1}, d_{j2}, ..., d_{jn})$  j = 1, 2. Where  $d_{jk}$  is the demand for product,  $P_j$ , from retailer  $R_k$ , k = 1, 2, ..., n in the planning period. We assume that the total demand for products  $P_1$  and  $P_2$  from each retailer during a distribution cycle is a constant number C. That is,  $C = d_{1k} + d_{2k}$ , k = 1, 2, ..., n. The total demand  $\tau_j$  during the distribution cycle for product  $P_j$  is obtained from the following equations.

$$\tau_j = \sum_{k=1}^n d_{jk}, j = 1, 2 \tag{1}$$

$$p_i = \tau_i / n, j = 1, 2$$
 (2)

#### 2.2 Supplies from the Manufacturer

The manufacturer produces C units in every time period such that  $C = p_1 + p_2$ , where  $p_1$  and  $p_2$  are the average production rates of the two products as given by equation (2). The manufacturer, being the dominant partner, will set this production rate in the first period and will maintain it throughout the distribution cycle.

The distributor receives his supplies at a constant rate of  $p_j$ , j = 1, 2 units, in the Figure 1 example,  $p_1 = 63, p_2 = 37$  units in every time period s (s = 1, 2,..., n), but the demand from retailers does not necessarily match the supplies from the manufacturer. Therefore, the distributor may not be able to find a feasible schedule without incurring shortages even though the total supplies from the manufacturer to the distributor are equal to the total demand from the retailers for both the products. However, shortages or back-orders are not allowed and the total demand of a retailer should be satisfied in a single supply in the chosen time period. This

may require the distributor to carry some extra units in the beginning (period 0) to meet the demand from the retailers. Let  $I_{1,0} \ge 0$  and  $I_{2,0} \ge 0$  be the initial inventory for products j = 1,2. It has to be noted here that  $I_{j,0}$ , j=1, 2 depends on the distribution sequence as explained in the paragraph below.

**Example:** A five period (n = 5) problem is shown in Figure 1. Here  $p_1 = 63$ ,  $p_2 = 37$  and C =100. The demand from retailers are  $D1 = \{20, 36,$ 86, 81, 92} and D2 =  $\{80, 64, 14, 19, 8\}$ . The distributor has 5! sequences to choose from; given the manufacturers production schedule. Suppose he chooses the distribution sequence shown in Figure 1, i.e., v = (1, 3, 4, 2, 5), in which the retailer who is served in sequence position1 has a demand of 20 units and 80 units for products  $P_1$  and  $P_2$  respectively. The distributor will be short of 43 units for product  $P_2$  and will have an excess 43 units of  $P_1$  as he receives 63 units of  $P_1$  and 37 units of  $P_2$  from the manufacturer. Hence, he must have a beginning inventory of 43 units of  $P_2$  to avoid shortages. Similar observations can be made for retailers served in subsequent positions. This example shows that if the distribution and production do not match in terms of the required quantities of the two products, the distributor will end up carrying inventory.

The problem is formulated as a mixed integer program. The original formulation which is an assignment problem with side constraints is due to Manoj et al. (2008).

Let  $x_{rs} = 1$  if the distributor delivers to retailer r during period s.  $d_{j,r}$  is the demand for product j for retailer r. Note that, the objective function for the integer program is the total inventory holding cost calculated by the end of inventory

level in each period. The distributor has an available inventory of  $I_{j,0} \ge 0$ , j = 1, 2 for the two products at the beginning of period 1. Let us denote  $I_{j,s}$  to be the inventory at the end of period s. This when expressed in the average inventory level is given by the equation (14) to be developed in Section 3.

Minimize 
$$h_1 \sum_{s=1}^{n} I_{1s} + h_2 \sum_{s=1}^{n} I_{2s}$$

$$\sum_{s=1}^{n} x_{rs} = 1, \quad r = 1, ..., n$$
 (3)

$$\sum_{r=1}^{n} x_{rs} = 1, \quad s = 1, ..., n$$
 (4)

$$I_{1,s} = I_{1,s-1} + p_1 - \sum_{r=1}^{n} d_{1r} x_{rs}, \quad s = 1,...,n$$
 (5)

$$I_{2,s} = I_{2,s-1} + p_2 - \sum_{r=1}^{n} d_{2r} x_{rs}, \quad s = 1,...,n$$
 (6)

$$I_{1,s} \ge 0, \quad s = 1,...,n$$
 (7)

$$I_{2,s} \ge 0, \quad s = 1,...,n$$
 (8)

$$x_{rs} \in \{0,1\}, s = 1,...,n; r = 1,...,n$$
 (9)

Constraints (3) and (4) ensure that exactly one retailer is served in each period. Constraints (5) and (6) find ending inventory of products  $P_1$  and  $P_2$  in a period, s.

## 2.3. Inventory Levels

In period 1,  $p_j$  units of product  $P_j$  are delivered by the manufacturer to the distributor and the distributor supplies  $d_{j,v(1)}$  the retailer served in position 1 of his distribution sequence. Recall,  $d_{j,v(i)}$  is the demand of the retailer who is served in position i of the distribution sequence v. Therefore, the ending inventory of product  $P_j$ , j = 1, 2 at the end of period 1 will be  $I_{j,1} = I_{j,0} + p_j - d_{j,v(1)}$ , refer to Figure 2. Similarly, the ending

inventory of product  $P_j$ , at the end of period 2 will be  $I_{j,2} = I_{j,1} + p_j - d_{j,\nu(2)} = I_{j,0} + 2 p_j - d_{j,\nu(1)} - d_{j,\nu(2)}$ . Similarly, for period n, the ending inventory of product j, (j = 1, 2) will be:

$$I_{j,n} = I_{j,(n-1)} + p_j - d_{j,\nu(n)}$$

$$= I_{j,0} + np_j - \sum_{s=1}^n d_{j,\nu(s)}$$
(10)

It may be noted that  $I_{j,n} = I_{j,0}$  since  $np_j = \sum_{s=1}^n d_{j,\nu(s)}$ 

# 2.4. Inventory Holding Cost

Let the distributor's inventory holding costs be  $h_1$  and  $h_2$  per time period (t) per unit for the two products respectively. The inventory holding cost is calculated based on the average inventory in a given period. Let  $I_j$  be the total average inventory of product  $P_j$  for a distribution cycle consisting of n periods, i.e.,  $I_j$  is the summation of the average inventory of all n periods. Moreover, let  $\Pi_j$  be the inventory holding cost for product j for the n periods, where,  $\Pi_j = h_j I_j$ ,

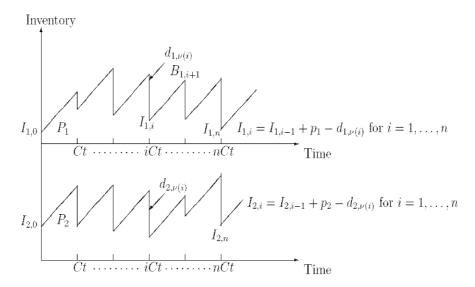
 $j = 1, 2. I_j$  is found as discussed in the Lemma given below. It may be assumed, without loss of generality, that  $h_1 \ge h_2$ .

**Lemma 1** For the manufacturer's preferred production rate of  $p_j$ , j = 1, 2 in every period, the distributor's total average inventory for n periods is given by:

$$I_{j} = nI_{j,0} + \frac{n(n+2)}{2} p_{j}$$

$$-h_{j} \sum_{s=1}^{n} (n+1-s) d_{j,\nu(s)}, \quad j = 1, 2$$
(11)

**Proof.** As discussed in the previous section and shown in Figure 2, the ending inventory level for period i when the initial inventory is  $I_{1,0}$  is  $I_{j,i} = I_{j,0} + iP_j - \sum_{k=1}^i d_{j,\nu(k)}$ ,  $j=1, 2, i=1, 2, \ldots, n$  as discussed above. Correspondingly the beginning inventory for the period i is given by  $B_{j,i} = I_{j,0} + (i-1)p_j - \sum_{k=1}^{i-1} d_{j,\nu(k)}$ , j=1, 2. The average inventory in any period i is given by



**Figure 2** Inventory level for a distribution sequence *v* 

$$\begin{split} \frac{(I_{j,i} + B_{j,i})}{2} &= I_{j,0} + \frac{(2i-1)p_j}{2} \\ &- \sum_{k=1}^i d_{j,v(k)} + \frac{d_{j,v(i)}}{2} \end{split}$$

Hence the total average inventory for the n periods is given by,

$$\begin{split} I_{j} &= nI_{j,0} + \sum_{i=1}^{n} \frac{(2i-1)}{2} p_{j} \\ &- \sum_{i=1}^{n} \sum_{k=1}^{i} d_{j,v(k)} + \sum_{i=1}^{n} \frac{d_{j,v(i)}}{2} \\ I_{j} &= nI_{j,0} + \frac{n^{2}}{2} p_{j} - \sum_{s=1}^{n} (n+1-s)d_{j,v(s)} + \frac{np_{j}}{2} \\ I_{j} &= nI_{j,0} + \frac{n(n+1)}{2} p_{j} - \sum_{s=1}^{n} (n+1-s)d_{j,v(s)} \end{split}$$

Hence the total average inventory holding costs for product j from period 1 to n denoted by  $\Pi_i$  is given by:

$$\Pi_{j} = h_{j} \left[ nI_{j,0} + \frac{n(n+1)}{2} p_{j} \right]$$

$$-h_{j} \left[ \sum_{s=1}^{n} (n+1-s) d_{j,\nu(s)} \right], \ j = 1, 2 \quad (12)$$

## 3. Calculating Inventory Cost

In this section we present the objective function and inventory constraints. For the ease of analysis we express  $p_2$  and  $d_{2,v(s)}$  in equation (12) in terms of  $p_1$  and  $d_{1,v(s)}$  respectively, i.e.  $p_2 = C - p_1$  and  $d_{1,v(s)} = C - d_{2,v(s)}$ .

#### 3.1 Objective Function

The objective is to find the values of  $I_{j,0}$ , j = 1, 2 and a distribution sequence  $v^* = (v(1), v(2),..., v(s),..., v(n))$ , recall v(s) is the retailer who is served in time period s, to minimize the total inventory holding cost under the constraints

imposed by the manufacturer. Let  $\Pi$  be the total inventory holding cost for the two products together, from period 1 to period n. The objective function is to minimize:

$$\Pi = \sum_{j=1}^{2} h_{j} \left[ nI_{j,0} + \frac{n(n+1)}{2} p_{j} \right] - h_{j} \left[ \sum_{s=1}^{n} (n+1-s) d_{j,\nu(s)} \right]$$
(13)

The objective function as stated in equation (13) can be rewritten as (see Appendix A for details):

$$\Pi = n[h_1 I_{1,0} + h_2 I_{2,0}] - p_1 (h_1 - h_2) \frac{n(n+1)}{2} + (h_1 - h_2) \sum_{s=1}^{n} s d_{1,\nu(s)}$$
(14)

Let us denote:

$$A = nh_1I_{1.0}$$

$$B = nh_2I_{2,0}$$

$$X = -\frac{p_1}{2} [n(n+1)(h_1 - h_2)]$$

And

$$Y = (h_1 - h_2) \sum_{s=1}^{n} s d_{1,\nu(s)}$$

The objective is, therefore, to minimize

$$\Pi = A + B + X + Y \tag{15}$$

It may be noted that in equation (15), X is a constant for a given problem. We observe from equation (15) that sequencing retailers in the nonincreasing order of the demand of product  $P_1$  will reduce the Y part of the inventory cost, if  $h_1 > h_2$ . However, this may result in an increase of the initial inventory cost part of the objective function (A and B), as shown later. Therefore, finding a sequence that minimizes the sum of costs Y, A and B is not straight forward. Based on the above expressions, we will identify some properties of the problem in the next section that

will be used in developing heuristic algorithms.

#### 3.2 Constraints

Consider a distribution sequence v = (5, 3, 4, 1, 2) of the example problem in Section 2.2. In this sequence the minimum initial inventory required to satisfy the demand for retailer 1 is (92-63) = 29, while that for satisfying demands for retailer up to the second position is  $(92 + 86 - 2 \times 63 = 52)$ . Let us denote the minimum inventory required to satisfy demand of retailer up to sequence position s as  $I_{j,0,s}$  j = 1, 2, s = 1, 2, ..., n. For any position s in a distribution sequence the following inventory constraints must be satisfied.

$$I_{1,0,s} + sp_1 - \sum_{q=1}^{s} d_{1,\nu(q)} \ge 0, \ s = 1, 2, ..., n$$
 (16)

$$I_{2,0,s} + sp_2 - \sum_{q=1}^{s} d_{2,\nu(q)} \ge 0, \ s = 1, 2, ..., n$$
 (17)

The minimum values of  $I_{1,0,s}$  and  $I_{2,0,s}$  to satisfy constraints (16) and (17) are given by equations (18) and (19) respectively by changing inequalities into equalities.

$$I_{1,0,s} = \sum_{q=1}^{s} d_{1,\nu(q)} - sp_1, \ s = 1, 2, ..., n$$
 (18)

$$I_{2,0,s} = \sum_{q=1}^{s} d_{2,\nu(q)} - sp_2, \ s = 1, 2, ..., n$$
 (19)

It may be noted that  $I_{1,0,s}$  and  $I_{2,0,s}$  may take both positive and negative values. A negative value means that no beginning inventory is required to satisfy the equation.

Let  $\Delta_{1,0}$  and  $\Delta_{2,0}$  be the minimum beginning inventory levels that will satisfy all equations (18 and 19) for the two products  $P_1$  and  $P_2$  respectively, where,

$$\Delta_{1,0} = \max \left\{ (I_{1,0,s}, 0), s = 1, 2, \dots, n \right\}$$
 (20)

$$\Delta_{2,0} = \max \left\{ (I_{2,0,s}, 0), s = 1, 2, \dots, n \right\}$$
(21)

 $\Delta_{j,0,j} = 1,2$  will be called the binding value. It may be noted that  $\Delta_{j,0} \ge 0$ .

# 4. Problem Properties

The problem defined by the objective function (equation (15)) and constraints (16) and (17) can be formulated as a mixed integer program as given in Section 2.2. However, finding an optimal sequence is NP-hard as shown by Manoj et al. (2008). Therefore, we will develop heuristic algorithms to solve this problem. The heuristic algorithms are based on the analysis of the structure of the problem. The structural properties of the problem are stated in the form of several lemmas as given below.

**Lemma 2** In a distribution schedule v that orders the retailers in nonincreasing order of demand of product  $P_1$  that is if  $d_{1,v(s)} \ge d_{1,v(s+1)}$ , s = 1, 2,..., n-1, then  $\Delta_{2,0} = 0$ . Similarly, if the distribution schedule v is to order the retailers in nondecreasing order of demand of product  $P_1$  that is if  $d_{1,v(s)} \le d_{1,v(s+1)}$ , s = 1, 2,..., n-1, then  $\Delta_{1,0} = 0$ .

**Proof.** From equation (19) we know that for any s, s = 1,..., n,

$$\begin{split} I_{2,0,s} &= \sum_{q=1}^{s} d_{2,\nu(q)} - sp_2 \\ &= \sum_{q=1}^{s} d_{2,\nu(q)} - s \left[ \frac{\sum_{q=1}^{n} d_{2,\nu(q)}}{n} \right] \\ &= \sum_{q=1}^{s} d_{2,\nu(q)} - s \left[ \frac{\sum_{q=1}^{s} d_{2,\nu(q)} + \sum_{q=s+1}^{n} d_{2,\nu(q)}}{n} \right] \end{split}$$

$$= \frac{n-s}{n} \sum_{q=1}^{s} d_{2,\nu(q)} - \frac{s}{n} \sum_{q=s+1}^{n} d_{2,\nu(q)}$$

Since  $d_{1,\nu(s)} \ge d_{1,\nu(s+1)}$ , we have  $d_{2,\nu(s)} \le d_{2,\nu(s+1)}$ , s = 1, 2, ..., n-1.

Now consider the case when s = n - 1. Substituting in the above we get:

$$I_{2,0,n-1} = \frac{1}{n} \sum_{q=1}^{n-1} d_{2,\nu(q)} - \frac{n-1}{n} d_{2,\nu(n)}$$

$$I_{2,0,n-1} = \frac{1}{n} \left\{ \tau - d_{2,\nu(n)} \right\} - \frac{n-1}{n} d_{2,\nu(n)}$$

$$I_{2,0,n-1} = \frac{1}{n} \{ \tau - nd_{2,\nu(n)} \}$$

Since  $d_{2,\nu(n)}$  is the largest demand we get  $\tau \le n$   $d_{2,\nu(n)}$ . The same result can be obtained for other s. Hence  $I_{2,0,s} \le 0 \ \forall \ s$ . Since  $\Delta_{2,0} = \max \{(I_{2,0,s}, 0), \ \forall \ s\}$ , we have  $\Delta_{2,0} = 0$ .

Similarly we can show that  $\Delta_{1,0} = 0$  if  $d_{1,\nu(s)} \le d_{1,\nu(s+1)}$ , s = 1, 2, ..., n-1. Hence the proof.

**Lemma 3** For  $h_1 > h_2$ , the term Y in the objective function (equation(15)) is minimized by ordering retailers in nonincreasing order of demand of product  $P_1$ ; that is  $d_{1,\nu(s)} \ge d_{1,\nu(s+1)}$ , s = 1, 2, ..., n-1.

**Proof.** Since  $Y = \sum_{s=1}^{n} s d_{1,\nu(s)}$ , ordering retailers in the nonincreasing order of product  $P_1$  demand will minimize Y and vice-versa.

#### 4.1 Problem Properties

From Lemmas 2 and 3 we know, for example, if retailers are sequenced in the non increasing order of demand size, then terms B and Y decrease but according to Lemma 2 it will increase term A. In order to exploit the properties of the problem further, we interchange two retailers in a given sequence and study the behavior of the cost function.

Consider two retailers in positions s and (s+1) in any sequence v where  $d_{1,v(s)} > d_{1,v(s+1)}$ . If the positions of these two retailers in the sequence are interchanged then the following Observations are made.

**Observation 1** The minimum beginning inventory  $I_{1,0,s}$  required to satisfy the constraint for period s for product  $P_1$  will decrease by an amount  $(d_{1,\nu(s)} - d_{1,\nu(s+1)})$ .

**Proof.** Let  $K = \sum_{q=1}^{s-1} d_{1,\nu(q)}$  and let  $d_{1,\nu(s)}$  and  $d_{1,\nu(s+1)}$  be the demand of the retailers served at the periods s and (s+1),  $(s+1) \le n$ , respectively in the sequence  $\nu$ .

$$I_{1.0.s} = K + d_{1.v(s)} - sp_1 (22)$$

$$I_{1,0,s+1} = K + d_{1,\nu(s)} + d_{1,\nu(s+1)} - (s+1) p_1$$
 (23)

After interchange the corresponding values are:

$$I'_{1,0,s} = K + d_{1,\nu(s+1)} - sp_1 \tag{24}$$

$$I'_{1,0,s+1} = K + d_{1,\nu(s+1)} + d_{1,\nu(s)} - (s+1)p_1$$
 (25)

Note that all other  $I_{1,0,j}$ ,  $j=1, 2,..., n-\{s, (s+1)\}$  values will remain unaffected after this interchange. The decrease in  $I_{1,0,s}$  as a result of the interchange is given by  $I_{1,0,s} - I'_{1,0,s}$  which is equal to  $d_{1,\nu(s)} - d_{1,\nu(s+1)}$ . Since  $d_{1,\nu(s)} > d_{1,\nu(s+1)}$ ,  $I_{1,0,s}$  decreases as a result of this interchange. Hence the proof.

**Observation 2** The minimum beginning inventory  $I_{1,0,(s+1)}$  required to satisfy the constraint for period (s+1) for product  $P_1$  will remain unchanged due to the interchange of  $d_{1,\nu(s)}$  and  $d_{1,\nu(s+1)}$ .

**Proof.**  $I_{1,0,(s+1)}$  before and after the interchange are given by equations (23) and (25) respectively, which are the same. Hence the proof.

**Observation 3** The value of  $\Delta_{1,0}$  may either

decrease or remain unchanged after the interchange.

**Proof.** We know from Lemma 2,  $I_{1,0,(s+1)}$  remains unchanged. Let  $\delta_k$  represents the maximum  $I_{1,0,j}$ ,  $j = \{1,2,...,n\} - \{s\}$ . Note that  $\delta_k$  will remain unchanged after the interchange. Let  $\delta_s$  represents  $I_{1,0,s}$  after the interchange.

$$\Delta_{1,0} = \max \left\{ \delta_k, \delta_s \right\}$$

From Lemma 1 we know that  $\delta_s$  decreases. If  $\delta_k \geq \delta_s$  then  $\Delta_{1,0}$  remains unchanged, if  $\delta_k < \delta_s$  then  $\Delta_{1,0}$  decreases.

**Observation 4** The minimum beginning inventory  $I_{2,0,s}$  required to satisfy the constraint for period s for product  $P_2$  will increase by an amount  $d_{1,\nu(s)} - d_{1,\nu(s+1)}$  which is also equal to  $d_{2,\nu(s+1)} - d_{2,\nu(s)}$ .

**Proof.** The corresponding equations  $I_{2,0,s}$  for the product 2 before and after the interchange are given below. Here  $K = \sum_{q=1}^{s-1} d_{2,\nu(q)}$ 

$$I_{2,0,s} = K + d_{2,\nu(s)} - sp_2 (26)$$

$$I_{2,0,s} = K + C - d_{1,\nu(s)} - sp_2$$
 (27)

$$I_{2,0,s+1} = K + 2C - d_{1,\nu(s)} - d_{1,\nu(s+1)} - (s+1) p_2$$
(28)

After the interchange:

$$I'_{2,0,s} = K + C - d_{1,\nu(s+1)} - sp_2$$
 (29)

$$I'_{2,0,s+1} = K + 2C - d_{1,\nu(s+1)} - d_{1,\nu(s)} - (s+1)p_2$$
(30)

Here  $I_{2,0,s} - I'_{2,0,s} = d_{1,\nu(s+1)} - d_{1,\nu(s)}$ . Since  $d_{1,\nu(s)} > d_{1,\nu(s+1)}$ , we have  $I_{2,0,s} - I'_{2,0,s} < 0$ . A negative decrease implies a positive increase. Hence the proof.

**Observation 5** The minimum beginning inventory  $I_{2,0,(s+1)}$  required to satisfy the constraint for period (s+1) for product  $P_2$  will remain unchanged due to the interchange of  $d_{1,v(s)}$ 

and  $d_{1,\nu(s+1)}$ .

**Proof.** Equations (28) and (30) represent the  $I_{2,0,(s+1)}$  values before and after the interchange respectively. They are equal and the proof for the Observation.

**Observation 6** The value of  $\Delta_{2,0}$  may either increase or remain unchanged after the interchange.

**Proof.** As before,  $\delta_k$  represents the maximum  $I_{2,0,j}$ ,  $j = \{1,2,...,n\} - \{s\}$ , Let  $\delta_s$  represents  $I_{2,0,s}$  after the interchange.

$$\Delta_{2,0} = \max \left\{ \delta_k, \delta_s \right\}$$

We know from Observation 4 that  $\delta_s$  increases. If  $\delta_k \ge \delta_s$  then  $\Delta_{2,0}$  remains unchanged, if  $\delta_k < \delta_s$ , then  $\Delta_{2,0}$  increases.

**Observation 7** The interchange of  $d_{1,\nu(s)}$  and  $d_{1,\nu(s+1)}$  may decrease term A in the objective function or it may remain unchanged.

**Proof.** From Lemma 3, we know that,  $\Delta_{1,0}$ , which is the minimum inventory to satisfy all  $I_{1,0,s}$  equalities may decrease; the proof follows.

**Observation 8** The term B in the objective function may increase or remain unchanged when  $d_{1,\nu(s)}$  and  $d_{1,\nu(s+1)}$  are interchanged.

**Proof.** Follows from Observation 6. An argument similar to Observation 7 applies here also.

**Observation 9** The term Y in the objective function will increase due to the interchange of  $d_{1,\nu(s)}$  and  $d_{1,\nu(s+1)}$ .

**Proof.** Follows from Lemma 3.

These properties are used to develop a heuristic (H2) which is given in the following section

## 5. Heuristic Algorithm

We propose two heuristics, H1 and H2, for

finding a good solution to the problem. Further, these solutions are used in the initial population of Genetic Algorithm which uses knowledge based crossover proposed by Aggarwal et.al (1997), to obtain good solutions quickly.

H1 is a greedy heuristic and H2 is a pairwise interchange heuristic. However due to the combinatorial nature of the problem, the proposed heuristics many times fail to explore the solution space for global minima, hence we use Genetic Algorithm (GA) to conduct a thorough search of the solution space. The knowledge based crossover helps to improve the quality of offsprings produced. These techniques are found to be effective in getting a good solution quickly.

#### 5.1 Heuristic H1

H1, constructs the distribution sequence by adding retailers one after another sequentially. For an empty sequence position (period in the problem), it chooses the retailer which gives the minimum inventory value for that period. The heuristic terminates when all retailers are sequenced. Before we present the heuristic, the notations used in it are explained in the following paragraph.

list is the set of retailer demands (of product  $P_1$ ) that are scheduled, whereas  $D_1$  – list is the set of unscheduled retailer demands. Let  $I_{j,k}$  be the inventory at the end of iteration k for product j and  $E_{j,k}(d_{1,i})$  be the ending inventory in the iteration k given that retailer i from  $D_1$  – list is scheduled. Here iteration involves selecting retailers based on the end of period inventory value in that period. Note that  $d_{2,i} = C - d_{1,i}$ . H1 schedules the retailer with minimum demand first (Step 1). The sequence is updated with a

retailer from  $D_1$  – list. The retailer appended will give the minimum inventory for that sequence position (Step 5). In a period k while doing the iteration, the end-of-period inventory value for a retailer is negative (shortage) for that position then the absolute value of the shortage will be factored by (k-1) (Step 3). This is done because shortage is not allowed, and the initial inventory should be increased by the shortage amount so that the distributor can make supply for the retailer. The selection will be done based on the new value. The heuristic is given below:

**Step 1**: list = 0; fix the retailer with the minimum demand for the product 1 first in the sequence, add the minimum demand  $(d_{1,1})$  to the *list*.

$$\begin{split} I_{1,0} &= \max \left\{ \left( d_{1,1} - p_1 \right), 0 \right\}, \\ I_{2,0} &= \max \left\{ \left( C - d_{1,1} \right) - p_2 \right), 0 \right\}, \\ I_{1,1} &= I_{1,0} + p_1 - d_{1,1} \ , \ I_{2,1} = I_{2,0} + p_2 - (C - d_{1,1} \ ) \\ \text{set } k = 1. \end{split}$$

**Step 2:**  $k \leftarrow k+1$ , Find  $E_{1,k}(d_{1,i}) = I_{1,(k-1)} + p_1 - d_{1,i}$ , and  $E_{2,k}(d_{1,i}) = I_{2,(k-1)} + p_1 - (C - d_{1,i})$  for all  $d_{1,i}$  in  $D_1 - list$ .

**Step 3:** If any  $E_{j,k}(d_{1,i}) < 0$ ,  $i = 1, 2, ..., |D_1 - list|$ , j = 1, 2;  $E_{j,k}(d_{1,i}) = (k-1)|E_{j,k}(d_{1,i})|$ .

**Step 4:** Calculate  $I_k(d_{1,i}) = h_1 E_{1,k}(d_{1,i}) + h_2 E_{2,k}(d_{1,i})$  where  $i = 1, 2, ..., |D_1 - list|, d_{1,i} \in D_1 - list$ , go to Step 5.

**Step 5:** Find min {  $I_k(d_{1,i})$ ,  $i = 1, 2, ..., |D_1 - list |$ } and let the corresponding demands for the two products be  $(d_{1,i}, (C - d_{1,i}))$ , and let  $d_{1,z} = d_{1,i}$  and  $d_{2,z} = (C - d_{1,i})$ . If necessary, break tie arbitrarily. **Step 6:** If any  $E_{j,k}(d_{1,z}) < 0$ , i = 1, 2, then  $I_{j,i} \leftarrow I_{j,i} + |E_{j,k}(d_{1,z})|$ , i = 0, 1, 2, ..., (k-1), j = 1, 2, update list with  $d_{1,z}$ . Find  $I_{j,k} = I_{j,(k-1)} + p_j - d_{j,z}$ , j = 1,2.

**Step 7:** If k=n, Stop, v = list; else go to Step 2.

Table 1 shows the heuristic applied to the example problem given in Figure 1 with  $h_1$ =  $h_2$ =1. The Step 1 assigns the retailer with the smallest demand for product 1 first in the sequence. The set *list* now contains element  $d_{1,\nu(1)}$ =  $d_{1,1}$  = 20. The next step is to find  $E_{1,k}(d_{1,i})$  and  $E_{2,k}(d_{1,i})$  values for all elements in  $D_1$  – list. It is shown in column 3 of Table 1, while column 4 shows all  $I_k$  values. For example, refer to the row k = 3,  $(E_{1,3}(92), E_{2,3}(92))$  values corresponding to demand 92 are (-9,52). From Step 3, we get  $E_{1,3}$  (92) = (3-1)×|-9| = 18. Hence  $E_3(92)$ , corresponding to demand 92, now becomes 18+52 = 70 (shown in Column 4). The demand pair  $(d_{1,z}, d_{2,z})$  which corresponds to the minimum  $I_k$  values are shown in column 5. Here in this example the complete sequence obtained is  $v^1 = (v(1), v(2), ..., v(n)) = (1, 3, 4, 2, 5).$ 

#### 5.2 Heuristic H2

This heuristic starts by sequencing retailers so that their demand for product  $P_1$  are in nonincreasing order. This will result in  $\Delta_{2,0}=0$ , as we know from Lemma 2. Since  $h_1 \geq h_2$  it is more important to minimize  $\Delta_{1,0}$  than  $\Delta_{2,0}$ , the heuristic is achieving the same. Interchanging adjacent retailers in the position with largest  $I_{1,0,s}$  may reduce  $\Delta_{1,0}$  as proven in Lemma 3, but  $\Delta_{2,0}$  may increase (Observation 6). The heuristic finds the binding value  $\Delta_{1,0}$  for a sequence and pairwise interchange adjacent demands in position with the largest  $I_{1,0,s}$  value until the terminal condition is encountered as explained in the following steps.

**Step 1:** Obtain the initial sequence  $v_0$  by ordering the retailers in the nonincreasing order of demand for product  $P_1$ , that is,  $v_0 = (v(1), v(2),...,v(n))$ , where,  $d_{1,v(1)} \ge d_{1,v(2)} \ge ... \ge d_{1,v(n)}$ .

Let  $\Pi_0$  be the inventory cost of this sequence. Set iteration number z = 1.

**Step 2:** Find the binding value  $\Delta_{1,0}$  and the sequence position (or positions) s at which this occurs. There may be several sequence positions that have the same value of  $I_{1,0,s}$ . Step 4 takes care of multiple sequence positions.

**Step 3:** Go to Step 7 if  $\Delta_{1,0} \le \varepsilon$  ( $\varepsilon$  is a small number, in our experiments we use  $\varepsilon = 3$ ), otherwise Go to Step 4.

**Step 4:** Interchange retailers in positions s and s+1. In case of multiple values of s, make all interchanges simultaneously.

**Step 5:** The interchange(s) in Step 4 give a new distribution sequence  $v_z$ , where z is the iteration number. Let  $\Pi_z$  be the cost of the sequence  $v_z$ .

**Step 6:** Set  $z \leftarrow z+1$ . Go to Step 2.

**Step 7:** STOP. A satisfactory sequence has been found.

**Step 8:** Select the sequence with minimum value of  $\Pi_{r}$ .

Once the distribution sequence for  $P_1$  is obtained then that for  $P_2$  will follow from our assumption that  $d_{2,v(s)} = C - d_{1,v(s)}$ .

An example for the first two iterations of the heuristic H2 is illustrated in Table 2. In this example, n=15,  $h_1=1.5$ ,  $h_2=1$ ,  $p_1=58$ . In the beginning (Step 1) retailers are arranged in the nonincreasing order of demand size.  $\Delta_{1,0}$  is given by position 7 i.e  $\Delta_{1,0}=I_{1,0,7}$ , so the retailers at positions 7 and 8 are mutually interchanged. These steps are repeated until  $\Delta_{1,0} \leq 3$ . Figure 3 shows  $\Delta_{1,0}$  and Inventory values as the iteration progresses.

As mentioned above in order to do a thorough search of the solution space we use Genetic Algorithm.

k	$D_1$ -list	$(E_{1,k},E_{2,k})$	$I_k$	$(d_{1,z},d_{2,z})$	list
2	{36,86,81,92}	(70, -27)	97	(86,14)	{20,86}
		(20,23)	43		
		(25,18)	43		
		(19,29)	43		
3	{36,81,92}	(47, -4)	55	(81,19)	{20,86,81}
		(2,41)	43		
		(-9,52)	70		
4	{36,92}	(29,14)	43	(36,64)	{20,86,81,36}
	, ,	(-27,70)	151		• • • • • •
5	{92}	(0,43)	43	(92,8)	{20,86,81,36,92}

Table 1 Heuristic H1 for the example problem

Table 2 Illustration of heuristic H2

Position	Itera	tion 1	Iter	ation 2
number, s	$d_{1,v(s)}$	$I_{1,0,s}$	$d_{1,\nu(s)}$	$I_{1,0,s}$
1	94	36	94	36
2	94	72	94	36
3	92	106	94	36
4	86	134	94	36
5	82	158	82	158
6	79	179	79	179
7	72	193	56	177
8	56	191	72	191
9	52	185	52	185
10	45	172	45	172
11	39	153	39	153
12	36	131	36	131
13	20	93	20	93
14	13	48	13	48
15	10	0	10	0
	Invento	ry = 3417	Invento	ory = 3380

# **Genetic Algorithm**

GA's are search techniques which are inspired by Darwin's theory of evolution. The evolution preserves the best sequences in the current generation and carries them to the next generation. In exploring the solution space, GA

uses randomization and directed smart search methods to obtain near optimal solutions.

In our problem, a chromosome is a distribution sequence. Each chromosome is characterized (merited) by its fitness value (here by its inventory value). GA emulates evolution

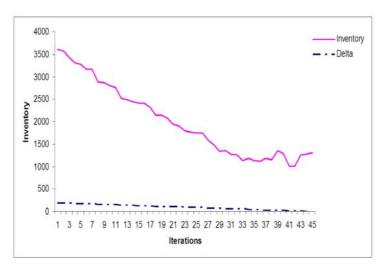


Figure 3 Evolution of delta and inventory

where each new iteration is a generation in the GA terminology. In GA environment a generation of chromosomes consists surviving chromosomes, usually the fitter ones from the previous generation, plus new child chromosomes produced in the current iteration through crossover. While GA is iterating, the population size is kept constant from one generation to the next. A crossover combines some features of two parent chromosomes and the child inherits characteristics from both parents. As GA iterates, fittest chromosomes reproduce and the least fit die. The initialization of GA is done as follows. We begin by randomly generating distribution sequences, for all values of n tested, from a U[10,95] distribution. Twenty three such sequences are generated, and the two heuristic (obtained from H1 and H2) sequences are added to make an initial population of 25.

#### Selection

We use Roulette wheel selection method for selecting parents for crossover. The selection is based on the fitness value; higher the fitness value of a chromosome, better are the chances that the chromosome gets selected. A random number is generated and the chromosome whose fitness value spans the random number is selected. Let's say, for example, the fitness value of the chromosome 1 and 2 are 0.1 and 0.3 respectively and let the random number generated has a value 0.15. In this case the chromosome 2 will span (0.1, 0.4) the random number and will get selected.

#### **Crossover and Mutation**

We use single point crossover method in which two offsprings are produced by mating two parents. The crossover point is randomly generated. Up to the crossover point one offspring will have the same sequence as that of one parent. Similarly the other child chromosome will get the same sequence as that of the second parent. To fill the remaining alleles (customer demand positions) in the first child chromosome we search the second parent's

chromosome from the left-most allele to the rightmost allele sequentially. When an unscheduled retailer demand is found, we copy this to the first offspring chromosome. We proceed this way until all remaining alleles are filled. The second child chromosome is produced like the first child chromosome using the first parent to fill all empty alleles after inheriting a part directly from the second parent up to the crossover point. Figure 4 shows a single point crossover method.

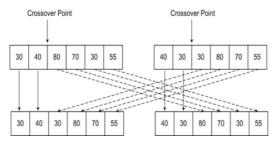


Figure 4 Illustration of single point crossover

With the aim to diversify the population, expecting quality child chromosomes, of the two child chromosomes thus produced through crossover, one will enter the population as it is, while the other child is optimized to improve the fitness value (Aggarwal et. al. 1997). The optimized crossover is done as explained below.

In the child chromosome to be optimized, identify all adjacent pairs such that  $d_{1,\nu(s)} > d_{1,\nu(s+1)}$ . Select each identified pair, one at a time do pairwise interchange and find the binding constraint  $\Delta_{1,0}$  for the resulting sequence. Undo the change and get back the original sequence. Similarly find  $\Delta_{1,0}$  for all other sequences which are obtained as a result of interchanging the demands which satisfies the condition  $d_{1,\nu(s)} > d_{1,\nu(s+1)}$  in the original sequence. Select that sequence which gives the minimum  $\Delta_{1,0}$  and

interchange the corresponding retailer pairs and an optimized child chromosome is obtained. In case of a tie in  $\Delta_{1,0}$  select that sequence which gives the minimum inventory cost. This optimization is based on Lemma 3.

An example of the optimizing process of child chromosome is shown in Table 3. Let the child chromosome be v = (45, 94, 36, 10, 94, 39,82, 79, 20, 13, 92, 86, 56, 72, 52) for product P<sub>1</sub> and that for product  $P_2$  be v = (55, 6, 64, 90, 6,61, 18, 21, 80, 87, 8, 14, 44, 28, 48). The inventory value for this child chromosome is 1638. The first column in Table 3 shows those demand pairs from the original sequence which satisfies the condition  $d_{1,\nu(s)} > d_{1,\nu(s+1)}$ . Column 2 gives  $\Delta_{1,0}$  values when the demands are interchanged. In this example interchanging 94 and 36 will give the minimum  $\Delta_{1.0}$  value. Interchange this demand pair and obtain the optimized child chromosome v = (45, 36, 94, 10,94, 39, 82, 79, 20, 13, 92, 86, 56, 72, 52). Sequence for  $P_2$  follows from that of  $P_1$ . The inventory cost corresponding to the new sequence obtained is 1487. We obtain an optimized child which has a lower inventory cost than the original one (1638). Optimized crossover not only diversifies the population but also helps in the fast convergence of the GA algorithm. The quality of the solution also improves as a result of this optimization process.

Mutation is done occasionally by adjacent pairwise interchange of alleles (randomly selected) in offspring chromosome. Whether a chromosome undergoes mutation or not depends on the mutation probability  $P_m$ . A random number is generated and if that number is less than  $P_m$ , then the offspring will undergo mutation.

Table 3 Optimized crossover

Demand pair	$\Delta_{1,0}$	Inventory cost
(94, 36)	15	1487.0
(36, 10)	23	1651.0
(94, 39)	23	1665.5
(82, 79)	23	1639.0
(79, 20)	23	1667.5
(20, 13)	23	1641.0
(92, 86)	23	1641.0
(86, 56)	23	1653.0
(72, 52)	23	1648.0

## 5.3 GA Algorithm

The following parameters are defined for the GA algorithm. 'GEN' represents the maximum number of generations in the GA, we use GEN = 10,000. The generation index is specified by variable  $g_n$ . PS represents the population size. The algorithm keeps track of the best sequence  $v^*$  obtained so far and records fitness value,  $f_i$ , of each sequence i obtained. Fifty best sequences will be fed to the following generation from the current generation. This way the best solution will be preserved from one generation to the next. The best sequence after GEN generations  $v^*$  will be the overall best sequence.

**Step 1:** Set n = 1.

**Step 2:** Generate 23 random sequences and add 2 heuristic solutions, obtained from H1 and H2, to make an initial population size of 25 (*PS*=25),  $v^* = v_1$  (arbitrary assignment).

**Step 3:** Evaluate each sequence  $v_i$ , i = 1, 2,..., PS, and find fitness value  $f_i (= \Pi_i / \sum_{j=1}^{PS} \Pi_j)$  of each of them, where  $\Pi_i$  is the inventory cost of the i<sup>th</sup> sequence. Order the chromosomes in the nonincreasing order of their fitness values.

**Step 4:** Select two parents as explained before from the ordered population of generation gn and crossover to produce two child chromomosomes as explained in Section 5. Mutate the child if the random number generated is less than  $P_m$ . Fifty chromosomes are produced by mating 25 parents. The best twenty five chromosomes (PS = 25) in generation  $g_n$  will be members of generation  $g_{(n+1)}$ .

**Step 5:** Sort all sequences in nonincreasing order of fitness values. Find the current best inventory value,  $v_n^*$ . If  $v_n^* < v_n^*$ , then set  $v_n^* = v_n^*$ . Preserve the top PS=25 members and delete the rest.

**Step 6:** If n = GEN go to step 8 or else go to Step 7.

**Step 7:** Set  $n \leftarrow n+1$  and go to Step 3.

**Step 8:**  $v^*$  is the best sequence obtained.

## 6. Computational Study

In this section we evaluate the performance of the heuristic presented in the previous section and establish the worth of doing the computational experiments and the justification to use them to solve industrial dimension problems.

The MIP given in Section 2.2 is solved for n = 15, 20, 25, 30 using CPLEX 8.01. The solution time depending on the problem instances ( $D_i$  and n) are between 0.11s to 3hrs with an optimality gap of around 1.6% on average, in an Intel Pentium Xeon Dual Processor, 2.4 Ghz computer. However for larger problems (n > 30) the optimality gap was around 15%. On the other hand the Genetic Algorithm took just 2 minutes to solve a 30 period problem in an Intel Pentium IV processor IBM machine with 512 MB RAM. CPLEX took more than

three hours to reach an optimality gap of 2%. In order to measure the performance of GA, we ran both GA and CPLEX for 2 minutes. The results are given in Table 6 (in Appendix B). Of the 60 instances tested, GA performed better in 24 instances while in 14 instances CPLEX performed better, and there was a tie in 22 instances. When GA performed better the solutions were on an average 3.96% better than CPLEX, on the other hand it was 0.93% when GA performed better. From the Table 6 it can be observed that for larger problem instances GA's performance was better. These factors further justified the usage of heuristics in our problem. Hence lesser computational time and good quality solutions from GA make the heuristic method more attractive than finding an optimal solution using CPLEX. The heuristic is tested for its performance for different set of  $D_i$  i=1, 2values (integer) generated randomly from U[10,90]. For a particular n five different sets of data are generated. The performance of the heuristics H1, H2 and GA for each data set is compared with the lower bound of the MIP solved using CPLEX. The Table 4, shows the performance (deviation from lower bound) of GA for different data sets. Columns H1, H2 and GA in Table 4 gives the inventory cost values of the best solution obtained from heuristics H1, H2 and Genetic Algorithm respectively. Column "lwrbd" is the optimal value of inventory obtained by solving the MIP using CPLEX. "% dvn from lwrbd" column shows the percentage deviation of the GA solution from the CPLEX solution. Except in one instance where the GA gives a solution which is 15.86% (shown bold in the Table 4) above the lower bound, the performance of the GA is very promising. The

worst case deviation is just 2.63%. Table 5 shows the overall performance of the heuristics which is given by the percentage deviation (average of 5 data sets tested for each n) from the lower bound. From Table 5 we can observe the worst case heuristic solution is around 27% above the lower bound. However, by conducting a thorough search of the solution space using GA with the initial population consisting of the sequences from H1 and H2, the quality of the solution improves considerably. In the worst case scenario the average deviation is around 3.5% above the lower bound.

#### 7. Model Extension

We show below as to how to extend the integer program model to three products types. Similarly, the model can be extended to any number of product types. It has to be noted that number of integer variables is n and it will not increase due to the increase in the number of product types. We can also extend our GA to any number of product types by modifying Step 3: Evaluate each distribution sequence  $v_i$ , i=1, 2, ..., PS, and find fitness value  $f_i$  (=  $\prod_i / \sum_{j=1}^{PS} \prod_j$ ) of each of

them, where  $\Pi_i$  is the inventory cost of the  $i^{th}$  sequence. Order the chromosomes in the nonincreasing order of their fitness values. The fitness function  $f_i$  for each sequence  $v_i$  will be calculated by considering inventories for all product types.

Minimize

$$h_1 \sum_{s=1}^{n} I_{1,s} + h_2 \sum_{s=1}^{n} I_{2,s} + h_3 \sum_{s=1}^{n} I_{3,s}$$

s.t.

$$\sum_{s=1}^{n} x_{rs} = 1, \quad r = 1, ..., n$$

$$I_{3,s} = I_{3,s-1} + p_3 - \sum_{r=1}^{n} d_{3r} x_{rs}, \quad s = 1, ..., n$$
(35)

$$\sum_{r=1}^{n} x_{rs} = 1, \quad s = 1, ..., n$$
(32) 
$$I_{1,s} \ge 0, \quad s = 1, ..., n$$

$$I_{2,s} \ge 0, \quad s = 1,...,n$$
 (37)

(38)

$$I_{1,s} = I_{1,s-1} + p_1 - \sum_{r=1}^{s-1} d_{1r} x_{rs}, \quad s = 1,...,n$$
 (33) 
$$I_{3,s} \ge 0, \quad s = 1,...,n$$

$$\sum_{r=1}^{n} x_{rs} = 1, \quad s = 1, ..., n$$

$$I_{1,s} = I_{1,s-1} + p_1 - \sum_{r=1}^{n} d_{1r} x_{rs}, \quad s = 1, ..., n$$

$$I_{2,s} \ge 0, \quad s = 1, ..., n$$

$$I_{3,s} \ge 0, \quad s = 1, ..., n$$

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$$I_{3,s}$$

Table 4 Performance of heuristics. Columns H1, H2, GA and lwrbd gives inventory cost in dollar values

	n	H1	H2	GA	lwrbd	% dvn from lwrbd	n	H1	H2	GA	lwrbd	% dvn from lwrbd
		660.0	660.0	660.0	660.0	0.00		920.0	920.0	920.0	920.0	0.00
<i>l</i> <sub>2</sub> _1		615.0	630.0	615.0	615.0	0.00		800.0	800.0	800.0	800.0	0.00
$h_1=1, h_2=1$	15	600.0	600.0	600.0	600.0	0.00	20	800.0	800.0	780.0	780.0	0.00
112-1		720.0	720.0	720.0	720.0	0.00		920.0	900.0	900.0	900.0	0.00
		630.0	660.0	630.0	630.0	0.00		800.0	960.0	800.0	800.0	0.00
		1450.0	1150.0	1125.0	1125.0	0.00		1170.0	1470.0	1350.0	1350.0	0.00
<i>l</i> <sub>2</sub> _1		1150.0	1125.0	1075.0	1075.0	0.00		1650.0	1410.0	1290.0	1290.0	0.00
$h_1=1, h_2=1$	25	975.0	1050.0	975.0	975.0	0.00	30	1140.0	1260.0	1170.0	1140.0	2.63
<i>n</i> <sub>2</sub> –1		1250.0	1150.0	1075.0	1075.0	0.00		1380.0	1410.0	1230.0	1230.0	0.00
		1125.0	1200.0	1000.0	1000.0	0.00		1470.0	1560.0	1200.0	1200.0	0.00
	15	766.0	810.0	759.0	747.0	1.61	20	1215.5	1111.5	1031.5	1030.0	0.15
<i>l</i> 1 5		707.5	795.0	703.0	700.0	0.43		915.5	993.0	901.5	896.0	0.61
$h_1=1.5, h_2=1$		795.0	721.0	689.5	682.0	1.10		893.5	965.0	890.5	885.5	0.56
<i>n</i> <sub>2</sub> –1		824.5	851.0	805.5	801.0	0.56		1308.5	1100.5	1034.0	1032.0	0.19
		734.0	811.0	730.0	717.0	1.81		995.0	1205.0	941.0	939.0	0.21
	25	1579.0	1395.0	1287.0	1270.0	1.34	30	2014.5	1793.5	1558.5	1553.0	0.35
1. 1.5		1303.0	1419.0	1218.0	1210.0	0.66		1559.5	1751.5	1470.0	1459.0	0.75
$h_1$ =1.5, $h_2$ =1		1235.5	1295.5	1137.0	1132.0	0.44		1392.5	1586.5	1332.0	1323.0	0.68
<i>n</i> <sub>2</sub> -1		1570.5	1391.5	1218.0	1213.0	0.41		2103.5	1720.0	1463.0	1463.0	0.00
		1227.0	1909.5	1195.5	1175.5	1.66		1493.0	1909.0	1406.5	1405.0	0.11
		872.0	947.0	834.0	834.0	0.00		1391.0	1303.0	1142.0	1142.0	0.00
1. 2		821.0	960.0	787.0	785.0	0.25		1055.0	1166.0	1008.0	992.0	1.61
$h_1=2, h_2=1$	15	852.0	842.0	764.0	764.0	0.00	20	1007.0	1130.0	992.0	991.0	0.10
<i>n</i> <sub>2</sub> -1		929.0	967.0	882.0	882.0	0.00		1573.0	1301.0	1170.0	1166.0	0.34
		857.0	947.0	804.0	804.0	0.00		1150.0	1440.0	1084.0	1084.0	0.00
		1765.0	1641.0	1435.0	1428.0	0.49		2259.0	2087.0	1809.0	1780.0	1.63
1 0	25	1426.0	1674.0	1354.0	1351.0	0.22		1739.0	2093.0	1642.0	1642.0	0.00
$h_1=2, h_2=1$		1421.0	1541.0	1285.0	1283.0	0.16	30	1585.0	1889.0	1511.0	1511.0	0.00
<i>n</i> <sub>2</sub> –1		1766.0	1633.0	1454.0	1255.0	15.86		2429.0	2030.0	1634.0	1634.0	0.00
		1608.0	1733.0	1372.0	1362.0	0.73		1801.0	2233.0	1632.0	1626.0	0.37

	$h_1 = 1,  h_2 = 1$			$h_1$	$= 1.5, h_2$	$_{2}=1$	$h_1 = 2,  h_2 = 1$		
	H1	H2	GA	H1	H2	GA	H1	H2	GA
15	0.0	1.44	0.00	2.59	9.42	1.10	6.82	14.69	0.05
20	0.96	4.51	0.00	10.77	12.54	0.35	14.15	18.02	0.41
25	12.93	8.31	0.00	14.30	23.06	0.90	19.74	23.36	3.49
30	18.74	14.67	0.00	18.38	21.78	0.38	19.43	26.26	0.40
			•			•			

**Table 5** Summary of performance of heuristics for different values of holding costs

#### 8. Conclusions

Finding an optimal schedule for the non-dominant partner in a two stage supply chain with a dominant upstream partner is NP-Hard. We attempt to solve large size problem in this research as the size of the integer programming problem that can be solved by CPLEX is limited. We propose two heuristics, a greedy heuristic and an interchange heuristic. The heuristics are based on the properties of the problem. Since the worst case average performance of these heuristics are around 20% we use sequences obtained from these heuristics in the initial population of Genetic Algorithm.

Further, we used optimized crossover technique to improve the quality of the solution and reduce the computation time. The performance of the Genetic Algorithm is extremely good for the problem sizes tested with an average gap of less than 3.5% from the

optimal solution for practical size problems.

Several important research issues remain open for future investigation. First, extending the model to allow for less than one truck load demand by the retailers. The less than one truck load demand may mean combining retailers' demand in to full truck load shipments which in turn requires routing decision. While we study the objective of minimizing inventory at the distributor, there are also other customer-related objectives that are relevant. These include minimization of the number of late deliveries and the total tardiness of the deliveries.

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# Appendix A

$$\begin{aligned} p_2 &= C - p_1, \quad d_{2,\nu(s)} = C - d_{1,\nu(s)} \quad \text{and} \quad \tau_1 = \sum_{s=1}^n d_{1,\nu(s)}, \text{ we can write the following:} \\ \pi_1 &= h_1 \left\{ nI_{1,0} + \frac{n(n+1)}{2} p_1 \right\} - h_1 \left\{ \sum_{s=1}^n (n+1-s) d_{1,\nu(s)} \right\} \\ &= h_1 \left\{ nI_{1,0} + \frac{n(n+1)}{2} p_1 \right\} - h_1 \left\{ (n+1) \tau_1 - \sum_{s=1}^n s d_{1,\nu(s)} \right\} \end{aligned}$$

## Appendix B

Table 6 Comparing the performance of GA and CPLEX after 2 minutes of CPU time

Table 6 Companing the performance of GA and CPLEA after 2 minutes of CPO time											
			G.4		CPLEX				CPLEX		
	n	lwrbd	GA		J Time=2 mins	n	lwrbd	GA		J Time=2 mins	
				Result	Opt. Gap				Result	Opt. Gap	
		660.0	660.0	660.0	0.59		920.0	920.0	920.0	4.72	
$h_1 = 1$ ,	15	615.0	615.0	615.0	0.86		800.0	800.0	800.0	1.78	
$h_1=1, h_2=1$		600.0	600.0	600.0	0.77	20	780.0	780.0	800.0	11.69	
<i>n</i> <sub>2</sub> -1		720.0	720.0	720.0	0.48		900.0	900.0	900.0	4.72	
		630.0	630.0	630.0	1.06		800.0	800.0	800.0	7.95	
		1125.0	1125.0	1125.0	30.91		1350.0	1350.0	1350.0	71.82	
$h_1 = 1$ ,		1075.0	1075.0	1075.0	13.06		1290.0	1290.0	1290.0	91.14	
$h_1=1, h_2=1$	25	975.0	975.0	975.0	9.53	30	1140.0	1170.0	1170.0	49.83	
$n_2 - 1$		1075.0	1075.0	1075.0	0.48		1230.0	1230.0	1260.0	42.86	
		1000.0	1000.0	1205.0	5.00		1200.0	1200.0	1200.0	27.5	
	15	747.0	759.0	747.0	1.07		1030.0	1031.5	1037.0	1.98	
L _1 5		700.0	703.0	700.0	1.29	20	896.0	901.5	900.0	1.61	
$h_1=1.5, h_2=1$		682.0	689.0	682.0	1.3		885.5	890.5	891.0	1.68	
$n_2$ -1		801.0	805.5	801.0	0.00		1032.0	1034.0	1032.0	2.57	
		717.0	730.0	717.0	1.53		939.0	941.0	950.0	17.52	
	25	1270.0	1287.0	1282.0	3.71		1553.0	1558.5	1580.0	52.10	
h _1 5		1210.0	1218.0	1230.0	2.93		1459.0	1470.0	1489.0	44.08	
$h_1=1.5,$		1132.0	1137.0	1132.0	3.68	30	1323.0	1332.0	1377.0	46.97	
$h_2 = 1$		1213.0	1218.0	1251.0	20.10		1463.0	1463.0	1491.0	34.74	
		1175.5	1195.5	1186.0	9.07		1405.0	1406.5	1480.0	35.57	
		834.0	834.0	834.0	1.92		1142.0	1142.0	1149.0	3.13	
1. 2		785.0	787.0	785.0	1.68		992.0	1008.0	992.0	1.41	
$h_1=2,$	15	764.0	764.0	764.0	2.07	20	991.0	992.0	1064.0	3.70	
$h_2 = 1$		882.0	882.0	882.0	1.25		1166.0	1170.0	1166.0	3.77	
		804.0	804.0	804.0	2.15		1084.0	1084.0	1092.0	10.90	
		1428.0	1435.0	1460.0	8.08		1780.0	1809.0	1850.0	46.98	
1. 2		1351.0	1354.0	1371.0	4.70	30	1642.0	1642.0	1715.0	48.43	
$h_1=2,$	25	1283.0	1285.0	1286.0	6.49		1511.0	1511.0	1835.0	49.66	
$h_2 = 1$		1255.0	1454.0	1406.0	19.32		1634.0	1634.0	1718.0	29.41	
		1362.0	1372.0	1381.0	13.97		1626.0	1632.0	1729.0	41.60	

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