



Single-machine scheduling problems with past-sequence-dependent delivery times

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ABSTRACT

We consider single-machine scheduling problems with past-sequence-dependent (p-s-d) job delivery times. The p-s-d delivery time is needed to remove any waiting time-induced adverse effects on the job's condition (prior to delivering the job to the customer) and it is therefore proportional to the job's waiting time. We show that single-machine scheduling problems with p-s-d delivery times and with either completion time-related criteria (such as the makespan or the total job completion time) or due date related criteria (such as the maximum lateness or the number of tardy jobs) can be solved by simple polynomial-time algorithms.

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1. Introduction

In a recent paper, Koulamas and Kyparisis (2008) considered single-machine scheduling problems in which a job's waiting time has an adverse effect on its condition. Koulamas and Kyparisis (2008) considered the case in which this adverse effect must be removed prior to the main processing of the job by performing a setup operation and introduced the concept of past-sequence-dependent (p-s-d) setup times. In this paper, we consider an alternative situation in which the waiting time-induced adverse effect does not impede the job's suitability to be processed by the machine even though this adverse effect must be removed prior to delivering the job to the customer. For example, an electronic component may be exposed to certain electromagnetic and/or radioactive fields while waiting in the machine's pre-processing area and regulatory authorities require the component to be "treated" (e.g., in a chemical solution capable of removing/neutralizing certain effects of electromagnetic/radioactive fields) for an amount of time proportional to the job's exposure time to these fields. This treatment can be performed after the component has been processed by the machine but before it is delivered to the customer so it can be delivered with a "guarantee". Such a post-processing operation is usually called the job "tail" or the job "delivery time". Unlike the traditional assumption of a job-specific constant delivery time in the scheduling literature, the above discussion justifies the assumption of a job delivery time proportional to the job's waiting time in order to model the mandated post-processing job "treatment".

It is of interest to notice that Browne and Yechiali (1990) incorporated the adverse effects of waiting into the job's main processing time by utilizing the concept of deteriorating job

processing times. Koulamas and Kyparisis (2008) incorporated the adverse effects of waiting into a pre-processing setup time by introducing the concept of past-sequence-dependent (p-s-d) setup times. In contrast, in the current paper, the adverse effects of waiting are incorporated into a post-processing operation by introducing the concept of past-sequence-dependent (p-s-d) delivery times. A formal definition of the problem will be presented later in this section.

Koulamas and Kyparisis (2008) showed that single-machine scheduling problems with completion time-related criteria and p-s-d setup times can be solved in $O(n \log n)$ time (where n is the number of jobs) by a sorting procedure. However, Biskup and Herrmann (2008) showed that single-machine scheduling problems with due date related criteria (such as the maximum lateness and/or the number of tardy jobs) and p-s-d setup times cannot be solved by simple procedures. The same argument holds true in the case of job deteriorating processing times. In this paper we show that in the presence of p-s-d delivery times, single-machine scheduling problems with either completion time-related criteria (such as the makespan and the total (average) job completion time) or due date related criteria (such as the maximum lateness, the maximum tardiness and the number of tardy jobs) can be solved by simple polynomial-time algorithms.

In order to formally define our problem, we consider a standard continuously available single-machine with a batch of n non-preemptive jobs available for processing at time zero. Let p_j , d_j denote the processing time and the due date, respectively, of job J_j , $j = 1, \dots, n$; also, let $J_{[j]}$, $p_{[j]}$, $d_{[j]}$ denote the job occupying the j position in the sequence, its processing time and its due date, respectively. The processing of job $J_{[j]}$ must be followed by a p-s-d delivery time $q_{[j]}$, which can be computed as

$$q_{[j]} = \gamma w_{[j]} = \gamma \sum_{i=1}^{j-1} p_{[i]}, \quad j = 2, \dots, n, \quad q_{[1]} = 0$$

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where $\gamma \geq 0$ is a normalizing constant and $w_{[j]}$ denotes the waiting time of job $J_{[j]}$. Observe that in a single-machine environment with a continuously available machine and all jobs simultaneously available at time zero, $w_{[1]}=0$ and $w_{[j]} = \sum_{i=1}^{j-1} p_{[i]}, j=2, \dots, n$.

It is assumed that the post-processing operation of any job $J_{[j]}$ modeled by its delivery time $q_{[j]}$ is performed “off-line”, consequently, it is not affected by the availability of the machine and it can commence immediately upon completion of the main operation resulting in $C_{[1]}=p_{[1]}+q_{[1]}=p_{[1]}$ and

$$C_{[j]} = w_{[j]} + p_{[j]} + q_{[j]} = \sum_{i=1}^{j-1} p_{[i]} + p_{[j]} + \gamma \sum_{i=1}^{j-1} p_{[i]} = (1 + \gamma) \sum_{i=1}^{j-1} p_{[i]} + p_{[j]}, \quad j = 2, \dots, n \tag{1}$$

where $C_{[j]}$ denotes the completion time of job $J_{[j]}$; C_j is defined analogously.

Using the standard three-field notation, our scheduling problem can be denoted as $1/q_{psd}/f(C_j)$ where $f(C_j)$ is a function of C_j . In this paper we will consider the minimization of the following five functions: the maximum completion time (make-span) $C_{\max} = \max_{j=1, \dots, n} \{C_j\}$, the total completion time $TC = \sum_{j=1}^n C_j$, the maximum lateness $L_{\max} = \max_{j=1, \dots, n} \{L_j\}$ where $L_j = C_j - d_j$ denotes the lateness of job J_j , the maximum tardiness $T_{\max} = \max_{j=1, \dots, n} \{T_j\}$ where $T_j = \max\{0, L_j\}$ denotes the tardiness of job J_j and the number of tardy jobs $\sum_{j=1}^n U_j$ where $U_j=1$ if $T_j > 0$ and $U_j=0$ if $T_j=0$ for $j=1, \dots, n$. The corresponding scheduling problems are denoted as $1/q_{psd}/C_{\max}, 1/q_{psd}/TC, 1/q_{psd}/L_{\max}, 1/q_{psd}/T_{\max}$ and $1/q_{psd}/\sum_{j=1}^n U_j$.

We close this section by mentioning that to the best of our knowledge, no paper with p-s-d delivery times has appeared in the literature till now. Past literature on scheduling problems with standard job-specific delivery times can be found in the survey of Lawler et al. (1993). Recent papers on scheduling problems with sequence-dependent setups include that ones by Kovács et al. (2009) and Tahar et al. (2006).

The rest of the paper is organized as follows: the $1/q_{psd}/C_{\max}$ and $1/q_{psd}/TC$ problems are studied in Section 2. The $1/q_{psd}/L_{\max}, 1/q_{psd}/T_{\max}$ and $1/q_{psd}/\sum_{j=1}^n U_j$ problems are studied in Section 3 and the conclusions of this research are summarized in Section 4.

2. The $1/q_{psd}/C_{\max}$ and $1/q_{psd}/TC$ scheduling problems

We first consider the $1/q_{psd}/C_{\max}$ problem. Let $P = \sum_{j=1}^n p_j$.

Proposition 1. The $1/q_{psd}/C_{\max}$ problem can be solved in $O(n)$ time.

Proof. It is clear from expression (1) that $C_{[j]} > C_{[j-1]}$; therefore,

$$C_{\max} = C_{[n]} = P + \gamma \sum_{i=1}^{n-1} p_{[i]} = P + \gamma(P - p_{[n]}) = (1 + \gamma)P - \gamma p_{[n]} \tag{2}$$

Since P is a constant, C_{\max} is minimized when $p_{[n]}$ is maximal. Consequently, any sequence with $p_{[n]} = \max_{j=1, \dots, n} \{p_j\}$ is optimal for the $1/q_{psd}/C_{\max}$ problem. Since the longest job can be identified in $O(n)$ time, the $1/q_{psd}/C_{\max}$ problem can be solved in $O(n)$ time as well.

We now turn our attention to the $1/q_{psd}/TC$ problem. As pointed out by a referee, the $1/q_{psd}/TC$ problem can be reduced to the

standard $1//TC$ problem (with no p-s-d delivery times) by observing from (2) that $C_{[j]} = (1 + \gamma) \sum_{i=1}^j p_{[i]} - \gamma p_{[j]}$. Therefore,

$$TC = \sum_{j=1}^n C_{[j]} = \sum_{j=1}^n \sum_{i=1}^j (1 + \gamma) p_{[i]} - \gamma \sum_{j=1}^n p_{[j]} = \sum_{j=1}^n \sum_{i=1}^j (1 + \gamma) p_{[i]} - \gamma P \tag{3}$$

Since γP is a constant, expression (3) has the same structure as the corresponding TC expression for an instance of the standard $1//TC$ problem with job processing times given as $p'_j = (1 + \gamma)p_j, j=1, \dots, n$. Consequently, the $1/q_{psd}/TC$ problem can be solved in $O(n \log n)$ time by implementing the shortest processing time (SPT) sequence.

3. The $1/q_{psd}/L_{\max}, 1/q_{psd}/T_{\max}$ and $1/q_{psd}/\sum_{j=1}^n U_j$ problems

In this section, we show that the $1/q_{psd}/L_{\max}, 1/q_{psd}/T_{\max}$ and $1/q_{psd}/\sum_{j=1}^n U_j$ problems can be converted to the corresponding standard problems (without p-s-d delivery times) by making the appropriate p_j, d_j transformations suggested by a referee. Specifically, in accordance with expression (2), $L_{[j]} = C_{[j]} - d_{[j]} = \sum_{i=1}^j (1 + \gamma) p_{[i]} - (d_{[j]} + \gamma p_{[j]})$ and $T_{[j]} = \max\{L_{[j]}, 0\}$; therefore, the substitutions of $p'_j = (1 + \gamma)p_j$ and $d'_j = d_j + \gamma p_j, j=1, \dots, n$ reduce the above $L_{[j]}, T_{[j]}$ expressions to the corresponding expressions without p-s-d delivery times. Consequently, the $1/q_{psd}/L_{\max}, 1/q_{psd}/T_{\max}$ and $1/q_{psd}/\sum_{j=1}^n U_j$ problems reduce to the corresponding $1//L_{\max}, 1//T_{\max}$ and $1//\sum_{j=1}^n U_j$ problems without p-s-d delivery times. As a result, the $1/q_{psd}/L_{\max}$ and $1/q_{psd}/T_{\max}$ problems can be solved in $O(n \log n)$ time by implementing the earliest due date (EDD) sequence on the d'_j due dates and the $1/q_{psd}/\sum_{j=1}^n U_j$ problem can be solved in $O(n \log n)$ time by implementing Moore (1968) Algorithm with processing times $p'_j = (1 + \gamma)p_j$ and due dates $d'_j = d_j + \gamma p_j, j=1, \dots, n$.

4. Conclusions

We considered single-machine scheduling problems with past-sequence-dependent (p-s-d) job delivery times. The p-s-d delivery time is needed to remove any waiting time-induced adverse effects on the job's condition prior to delivering it to the customer and it is therefore proportionate to the job's waiting time. We showed that the $1/q_{psd}/C_{\max}$ problem can be solved in $O(n)$ time and that the $1/q_{psd}/TC, 1/q_{psd}/L_{\max}, 1/q_{psd}/T_{\max}$ and $1/q_{psd}/\sum_{j=1}^n U_j$ problems can be reduced to the corresponding problems without p-s-d delivery times and subsequently solved in $O(n \log n)$ time.

The significance of our results is highlighted by observing that in the presence of p-s-d delivery times, single-machine scheduling problems with due date related criteria (such as maximum tardiness and/or the number of tardy jobs) can be solved by simple procedures which is not possible in the presence of p-s-d setup times or in the presence of waiting time-induced deteriorating job processing times. Consequently, whenever possible, it is beneficial to perform a post-processing operation to remove any waiting time-induced adverse effects rather than a comparable pre-processing operation (setup) because in the former case problems with not only completion time-related criteria but also problems with due date related criteria can be handled with relative ease.

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