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Supply chain scheduling: Just-in-time environment

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Abstract We study the benefits of coordinated decision making in a supply chain consisting of a manufacturer, a distributor, and several retailers. The distributor bundles finished goods produced by the manufacturer and delivers them to the retailers to meet their demands. The distributor is responsible for managing finished goods inventory. An optimal production schedule of the manufacturer, if imposed on the distributor, may result in an increased inventory holding cost for the distributor. On the other hand, an optimal distribution schedule of the distributor, if imposed on the manufacturer, may result in an increased production cost for the manufacturer. In this paper we develop mathematical models for individual optimization goals of the two partners and compare the results of these models with the results obtained for a joint optimization model at the system level. We investigate the computational complexities of these scheduling problems. The experimental results indicate that substantial cost savings can be achieved at the system level by joint optimization. We also study conflict and cooperation issues in the supply chain. The cost of conflict of a supply chain partner is a measure of the amount by which the unconstrained optimal cost increases when a decision is to be made under the scheduling constraint imposed by the other partner. We quantify these conflicts and show that the cost of conflicts are significant. We also show that a cooperative decision will generate a positive surplus in the system which can be shared by the two partners to make cooperation and coordination strategy more attractive.

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1 Introduction

The importance of effective *supply chain management* is increasing because of competition and growing business complexities. A harmonious relationship between various members of a supply chain is the key to the survival and growth of a supply chain. However, supply chains are susceptible to power plays where a dominant partner may dictate terms to other members, thereby improving his own performance but decreasing the overall performance of the supply chain. The supply chain members therefore need to coordinate and collaborate in their operations to achieve global optimization.

In this paper we study coordination issues between a distributor and a manufacturer in a two-stage supply chain. Specifically, we study the local optimization of production schedules by the manufacturer, local optimization of distribution schedules by the distributor, and the joint optimization of production and distribution schedules to improve overall system performance in a JIT environment. The mathematical modeling of coordinated production planning, scheduling, and distribution in supply chains is still a relatively new area of research. Some of the studies that are reported in this area are briefly discussed below.

In one of the early papers, Ow et al. (1988) study a multi-agent scheduling system, but have not provided any detailed evaluation of benefits arising out of cooperation. Blumenfeld et al. (1991), study a network with one manufacturing facility producing multiple products and serving several customers. Goods are shipped directly to the customers by the manufacturer. They analyze the trade off between manufacturing, transportation, and inventory costs. They compare total cost for independent and synchronized schedules and show that cost savings are significant for a synchronized system.

The general issues of coordination in supply chains have been studied and emphasized by several authors including Banker and Khosla (1995), Karmarkar (1996), and Munson et al. (1999). Thomas and Griffin (1996) review literature on coordinated planning in supply chains. They classify the literature into three operational models: buyer-vendor coordination (Anupindi and Akella 1993), production-distribution coordination (Chandra and Fisher 1994), and inventory-distribution coordination (Muckstadt and Roundy 1993). They also identify directions for future research in this area. Sarmiento and Nagi (1999) review models that explicitly address logistics in supply chains. They point to the need of integrating logistical functions into production functions such as inventory control.

More recently Lee and Chen (2001), Chang and Lee (2003), Hall and Potts (2003), Agnetis et al. (2006), Chen and Hall (2008), Chen and Vairaktarakis (2005), Li and Xiao (2004), and Dawande et al. (2006) have studied supply chains that include various aspects of production scheduling, distribution planning, inventory management, and customer service level under different scenarios. Lee and Chen (2001) include transportation constraints, such as time and capacity constraints, into scheduling decisions and provide efficient heuristics. Chen and Vairaktarakis (2005) study a production and distribution model with a focus on customer service level and distribution costs, and provide efficient algorithms and heuristics for computationally intractable problems. However, neither of these two papers study conflict and coordination issues in a production distribution system.

Hall and Potts (2003) and Agnetis et al. (2006) model a two stage supply chain system that involves a supplier, several manufacturers, and their downstream customers. The decisions by the supply chain partners in Hall and Potts (2003) include scheduling, batching and delivery. They consider several classical scheduling objectives and present models to minimize the overall scheduling and delivery cost. They show that a cooperative decision between supply chain partners leads to a reduction in the total system cost. The actual reduction may vary from 20% to 100% based on the objective function. The problem studied by Agnetis et al. (2006) includes an intermediate storage buffer of limited capacity to resequence jobs between two stages in the system. They develop models to minimize two objective functions: (i) total interchange (change in relative job positions in the ideal schedule for the member) cost, and (ii) total interchange cost plus work-in-process cost. Chen and Hall (2008) study conflict and cooperation issues in a supply chain in which a manufacturer is served by several suppliers. They consider the following two objectives: minimize total completion time and minimize maximum lateness. They study four scenarios with relatively different bargaining powers of supply chain members.

Chang and Lee (2003) study two different types of two stage scheduling problems. The first problem is equivalent to the classical two-machine flow shop problem. The second problem involves processing of jobs in a manufacturing system and then delivery to the customers in batches. They consider several objective functions that have been studied in the classical scheduling literature and compare the worst case individual stage performance with that of the overall system performance. Li and Xiao (2004) study coordination issues that arise when large production lots are split into smaller sub-lots in a multistage manufacturing system managed by the same firm. The objective of the paper is to facilitate lot-splitting decisions by various partners to achieve a system-wide optimal solution.

Dawande et al. (2006) study conflict and cooperation issues in a supply chain consisting of a manufacturer and a distributor. They study two problems in which the manufacturer's objective is to minimize unproductive time in both problems, whereas the distributor minimizes customer cost measures in one problem and minimizes inventory holding cost in the other. The supplier and the manufacturer may make individual decisions or cooperative decisions. The level of cooperation between the two partners and a combination of the objective functions lead to various problem scenarios. The authors show that significant cost savings can be achieved in cooperative decisions. The research in our paper is most closely related to their paper.

In this paper, motivated by the work of Blumenfeld et al. (1991), we study a two-stage supply chain consisting of a manufacturer, a distributor, and several retailers. The supply chain operates in a Just-in-Time environment. The manufacturer produces two similar products on a single production line and transports the products to a nearby warehouse. The distributor bundles the products at the warehouse based on the demand requirement of the two products provided by the retailers and delivers the bundled products to the retailers within a pre-specified planning horizon. An example of such a scenario may be found in automobile manufacturing plants in which closely related models of automobiles are manufactured sequentially on the same assembly line (Ohno 1988). For example, the Toyota manufacturing facility at Georgetown, Kentucky, sequentially produces its Camry (car) and Sienna (minivan) on the same production line called the Global Body Line (Gardner 1997). Other examples of automobile manufacturing facilities producing multiple products on the same assembly line include Nissan Integrated Manufacturing systems (NIMS) (Nissan News 2005) and Chrysler's Ontario plant (Industrial Engineer 2005).

In the two-stage supply chain under investigation in this paper, the distributor incurs the cost of maintaining inventories at the warehouse, whereas the manufacturer incurs costs in changing the production rate. The production rate is an indicator of the ratio in which the two products are produced in any given time period. Changing production rate affects the manufacturer's costs in several ways. It requires the manufacturer to recalculate the parts requirements, coordinate the changes with the suppliers of parts, change computer programs to reflect new production plans, etc. In addition, if the required skill levels are different for the two products the manufacturer may end up using highly skilled workers for jobs that could be done by lower skilled workers. All these costs are part of what we have termed as production rate change cost.

The manufacturer prefers a production schedule that minimizes the total cost of changing production rates. The optimal schedule for the manufacturer will be to produce at a constant rate throughout the planning horizon. However, the distributor will prefer a production rate schedule that will minimize his inventory holding costs. Dawande et al. (2006) study a similar problem in which the two products are produced on the same production line. In their problem, switching products incurs a setup cost, while in our paper, switching products in the production line is done instantaneously and it does not require any major setup changes. Thus, the setup cost is zero, but changing the production rate of products has a cost associated with it as mentioned above.

The rest of this paper is organized as follows: in Sect. 2, we provide the problem definition, the notations and the assumptions. The costs of conflict for the distributor and the manufacturer are defined and mathematically formulated in Sect. 3. Section 3 also includes mixed integer programming formulations for evaluating these conflicts. Section 4 investigates the computational complexities of the individual problems of the manufacturer and the distributor. Section 5 presents the mixed integer programming formulation for the system problem and analyzes the benefits of coordination between the manufacturer and the distributor. We also discuss how the manufacturer and the distributor can negotiate, coordinate, and implement their supply chain schedules in practice. In Sect. 6, results of computational studies are presented and the conflicts are quantified. Section 7 describes briefly the implementation process to achieve coordination. Finally, conclusions and directions for future research are discussed in Sect. 8.

2 Problem scenario

In the two stage supply chain problem analyzed in this paper, the manufacturer produces two closely related products, P_1 and P_2, sequentially on the same assembly line. The products are to be distributed to n retailers $R_i, i = 1, \dots, n$, by the distributor. The distributor uses trucks for delivery, and each has a fixed capacity of C units. The time to produce one unit of either P_1 or P_2 is t and the time required to produce one truckload of products, called the production period, is Ct. A complete distribution cycle consists of nCt time periods. The total production during the distribution cycle is equal to the total demand from all retailers. During each distribution cycle, the total demand from each retailer is C, and this demand must be met by sending one truckload to each retailer. Therefore, one of the retailers R_i is served during each production period Ct.

Let d_{ij} denote the demand of product j for retailer i, $i = 1, \dots, n$ and $j = 1, 2$. Also let D_j denote the demand for product P_j for n periods, i.e., $D_j = (d_{1j}, d_{2j}, \dots, d_{nj})$, $j = 1, 2$. For retailer R_i, the ratio of the demands for the two products is $r_{i1} : r_{i2}$. Thus, to satisfy the demand from retailer R_i, the quantity of product P_j, $j = 1, 2$, in the truck is $d_{ij} = C\frac{r_{ij}}{r_{i1}+r_{i2}}$, $j = 1, 2$. The total demand of both products is nC and the overall ratio of the demands for the two products is $\tau_1 : \tau_2$ during the distribution cycle. Note that

$$\frac{\tau_1}{\tau_2} = \frac{\sum_{i=1}^{n} d_{i1}}{\sum_{i=1}^{n} d_{i2}}. \tag{1}$$

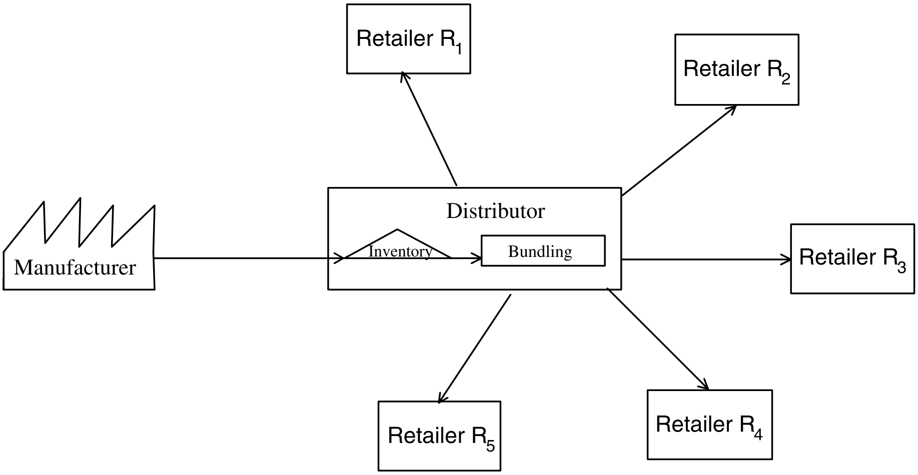


Fig. 1 Supply chain network

The assumption that each retailer receives one truckload of products is not restrictive. If a retailer requires multiple truckloads (say y), then the retailer can be treated as multiple (y) retailers, each with a demand of one truckload. The rate of production of each product P_j, $j = 1, 2$, if required, can be changed from one production period to the next period. The cost of changing the production rate from a period i to period $i+1$ is μ, $i = 1, \dots, n$. Let p_{ij} be the rate of production of product P_j for period i, $i = 1, \dots, n$. Let $\sigma = (p_{11}, p_{21}, \dots, p_{n1})$ denote the sequence of production rate of product P_1 throughout the distribution cycle. Note that $p_{i2} = C - p_{i1}$, $i = 1, \dots, n$. The manufacturer can set one production rate for all periods and incur a total rate change cost of only μ, i.e., for the rate change in the beginning of a cycle and the production rates will be as follows: $p_{11} = p_{21} = \cdots = p_{n1} = p_1$, where $p_1 = C\frac{\tau_1}{(\tau_1+\tau_2)}$ and $p_2 = C - p_1$.

Other notations used in this paper include the following: let $I_{j,s}$ be the inventory level of product P_j, $j = 1, 2$, at the end of period s. Initial inventory of product P_j is $I_{j,0}$, $j = 1, 2$. The inventory holding cost per production period for product P_j is h_j, $j = 1, 2$, per unit.

The manufacturer's problem is to minimize the total rate change cost per distribution cycle. Since the production cost during a distribution cycle is constant, minimizing the total manufacturing cost is equivalent to minimizing the total cost of changing production rate during the distribution cycle. For a given production rate sequence σ, let $S(\sigma)$ denote the corresponding manufacturer's cost. The distributor's problem is to find a distribution sequence $\nu(\sigma)$ that minimizes his inventory holding cost, $T(\sigma, \nu(\sigma))$. Note that $\nu(\sigma)$ is the sequence in which the retailers are served during the distribution cycle. The system problem is to minimize the total system cost, $S(\sigma) + T(\sigma, \nu(\sigma))$ over all production rate sequences σ and distribution sequences $\nu(\sigma)$.

Example problem A supply chain network with five retailers is shown in Fig. 1. During each of the five periods, the manufacturer produces a total of 100 units of P_1 and P_2. The total demand from each retailer is equal to the truck capacity of $C = 100$ units and each product has a total demand of 250 units from all retailers. Here $h_1 = h_2 = 1$, $D_1 = (30, 40, 80, 70, 30)$ and $D_2 = (70, 60, 20, 30, 70)$. A cost of $\mu = 25$ is required for changing the production rate from one period to another. This example will be referred to in later sections.

3 Cost of conflict

The cost of conflict of a supply chain partner is defined as the amount by which the unconstrained optimal cost increases when a decision is to be made under the scheduling constraint imposed by the other partner. We identify the cost of conflicts for the two partners in this section.

3.1 Distributor's conflict

The distributor's conflict arises when the manufacturer dominates, decides his schedule and imposes it on the distributor. The distributor has to find an optimal delivery sequence that minimizes his total inventory holding cost within the production schedule imposed by the manufacturer. The distributor's conflict, which is the percentage increase in cost of the distributor, is:

$$[T(\sigma^*, \nu(\sigma^*)) - T(\sigma(\nu^*), \nu^*)]100/T(\sigma(\nu^*), \nu^*), \tag{2}$$

where $\sigma(\nu^*)$ is the manufacturer's rate schedule preferred by the distributor so that the delivery sequence, ν^* is optimal. The manufacturer uses his optimal rate schedule σ^* and the distributor must determine his best schedule $\nu(\sigma^*)$, given the manufacturer's schedule σ^*. As can be seen later, there may be many combinations of sequences $(\sigma(\nu^*), \nu^*)$ which give optimal delivery sequence. We find the one that provides the minimum number of production rate changes in $\sigma(\nu^*)$.

The unconstrained optimal sequence for the manufacturer is found by minimizing the number of production rate changes. The manufacturer sets one production rate throughout the distribution cycle.

Theorem 1 *The manufacturer's problem is solved by a production rate sequence* $\sigma^* = (p_1, p_1, \dots, p_1)$, *where* $p_1 = C\frac{\tau_1}{(\tau_1+\tau_2)}$. *The total cost of production rate change is* μ.

Proof In this case, the manufacturer sets only one production rate throughout the distribution cycle incurring cost μ in the beginning. To satisfy the demand, the manufacturer must set the production rate for product P_1, $p_1 = C\frac{\tau_1}{(\tau_1+\tau_2)}$. Thus, the production rate for product P_2, $p_2 = C - p_1$. □

The manufacturer can always minimize his total cost by having only one production rate for each product, whichever distribution sequence is chosen by the distributor.While doing so may increase inventory costs for the distributor, the manufacturer is not responsible for these costs.

Finding $T(\sigma^*, \nu(\sigma^*))$ For a given σ^* the distributor has to identify $\nu(\sigma^*)$ and calculate $T(\sigma^*, \nu(\sigma^*))$. To determine a delivery schedule $\nu(\sigma^*)$ that will minimize the inventory cost for the distributor, we formulate a mixed integer program. The objective represents the total inventory holding cost as measured by the end of inventory level at each period for both products given the production rate sequence, $\sigma^* = (p_1, p_1, \dots, p_1)$, where $p_1 = C\frac{\tau_1}{(\tau_1+\tau_2)}$. However, before we provide the mixed integer program, we require the following expression for the distributor's inventory holding cost.

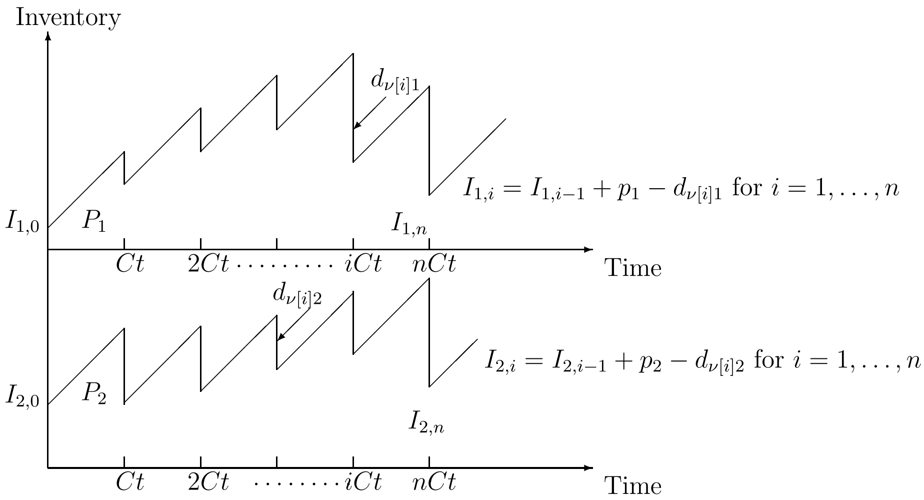


Fig. 2 Inventory level

Lemma 1 *Given production rate sequence*, $\sigma^* = (p_1, p_1, \ldots, p_1)$, *the distributor's inventory holding cost can be expressed as follows*:

$$T(\sigma^*, \nu(\sigma^*)) = \frac{n}{2}(p_1h_1 + p_2h_2) + \sum_{i=0}^{n-1}(I_{1,i}h_1 + I_{2,i}h_2).$$

Proof Figure 2 shows the inventory graph for both products in a distribution cycle. Initially, the inventory of product P_1 is $I_{1,0}$. Let $\nu[i]$ denotes the ith retailer served in the distribution sequence ν. At time Ct (at the end of period 1), the total inventory of product P_1 is $I_{1,1} = I_{1,0} + p_1 - d_{\nu[1]1}$, as retailer $R_{\nu[1]}$ is served in this period, and the inventory cost for this period is $\frac{1}{2}h_1(I_{1,0} + I_{1,0} + p_1)$. At time $2Ct$ (at the end of period 2), the total inventory of product P_1 is $I_{1,2} = I_{1,1} + p_1 - d_{\nu[2]1}$, as retailer $R_{\nu[2]}$ is served, and the inventory cost for this period is $\frac{1}{2}h_1(I_{1,1} + I_{1,1} + p_1)$, and so on. Finally, the inventory cost of product for nth period for P_1 is $\frac{1}{2}h_1(I_{1,n-1} + I_{1,n-1} + p_1)$. Let the inventory holding cost for period 1 to period n for product P_i is T_i. We thus have

$$T_1 = \sum_{i=1}^{n} \frac{1}{2}h_1(I_{1,i-1} + I_{1,i-1} + p_1) = \frac{n}{2}p_1h_1 + \sum_{i=0}^{n-1} I_{1,i}h_1. \tag{3}$$

Similarly, the inventory holding cost for period 1 to period n for product P_2 is

$$T_2 = \sum_{i=1}^{n} \frac{1}{2}h_2(I_{2,i-1} + I_{2,i-1} + p_2) = \frac{n}{2}p_2h_2 + \sum_{i=0}^{n-1} I_{2,i}h_2. \tag{4}$$

The result then follows from (3) and (4) as $T(\sigma^*, \nu(\sigma^*)) = T_1 + T_2$. □

Note that in the above lemma, the inventory term $\frac{n}{2}(p_1h_1 + p_2h_2)$ is a constant for any combination of sequences $(\sigma^*, \nu(\sigma^*))$. The constant term, $\frac{n}{2}(p_1h_1 + p_2h_2)$ can also be expressed as $\frac{1}{2}(h_1\sum_{i=1}^{n} d_{i1} + h_2\sum_{i=1}^{n} d_{i2})$.

For the production rate sequence $\sigma^* = (p_1, p_1, \dots, p_1)$, where $p_1 = C\frac{\tau_1}{(\tau_1+\tau_2)}$, finding an optimal sequence, $\nu(\sigma^*)$ for the distributor is shown to be strongly NP-hard in Sect. 4. The following mixed integer program formulation is devised to minimize the variable term of the distributor's inventory holding cost, $T(\sigma^*, \nu(\sigma^*))$. Let $x_{ri} = 1$ if the distributor delivers to retailer r during period i and 0 otherwise.

$$\text{Minimize } h_1 \sum_{i=1}^{n} I_{1,i} + h_2 \sum_{i=1}^{n} I_{2,i}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_{ri} = 1, \quad r = 1, \dots, n, \tag{5}$$

$$\sum_{r=1}^{n} x_{ri} = 1, \quad i = 1, \dots, n, \tag{6}$$

$$I_{1,i} = I_{1,i-1} + p_1 - \sum_{r=1}^{n} d_{r1} x_{ri}, \quad i = 1, \dots, n, \tag{7}$$

$$I_{2,i} = I_{2,i-1} + p_2 - \sum_{r=1}^{n} d_{r2} x_{ri}, \quad i = 1, \dots, n, \tag{8}$$

$$I_{1,n} = I_{1,0}, \tag{9}$$

$$I_{2,n} = I_{2,0}, \tag{10}$$

$$I_{1,i} \geq 0, \quad i = 1, \dots, n, \tag{11}$$

$$I_{2,i} \geq 0, \quad i = 1, \dots, n, \tag{12}$$

$$x_{ri} \in \{0, 1\}, \quad i = 1, \dots, n; \ r = 1, \dots, n. \tag{13}$$

Constraints (5) and (6) ensure that exactly one retailer is served in each period. Constraints (7) and (8) find ending inventory of products P_1 and P_2 in a period. Equations (9) and (10) ensure that the ending inventory in a distribution cycle will be equal to the starting inventory of the next cycle.

For the example problem given in Sect. 2, the distributor's problem is solved, given $\sigma^* = (50, 50, 50, 50, 50)$, using the mixed integer program formulation given above. The solution is presented in Fig. 3. The distributor's sequence, $\nu(\sigma^*) = (1, 4, 5, 3, 2)$ and the inventory holding cost, $T(\sigma^*, \nu(\sigma^*)) = 400$. Note that in the solution, the constant term, $\frac{n}{2}(p_1 h_1 + p_2 h_2) = 250$, the variable term, $\sum_{i=0}^{n-1}(I_{1,i} h_1 + I_{2,i} h_2) = 150$ and $S(\sigma^*) = 25$.

Finding $T(\sigma(\nu^*), \nu^*)$ To determine a production rate schedule, $\sigma(\nu^*)$ for the delivery schedule, ν^* that minimizes the distributor's unconstrained inventory cost, we formulate the following mixed integer program. Let $x_{ri} = 1$ if the distributor delivers to retailer r during period i. Let z_{ij} denotes the number of units of P_j produced in period i. The objective represents the total inventory holding cost for both products.

$$\text{Minimize } h_1 \sum_{i=1}^{n} I_{1,i} + h_2 \sum_{i=1}^{n} I_{2,i}$$

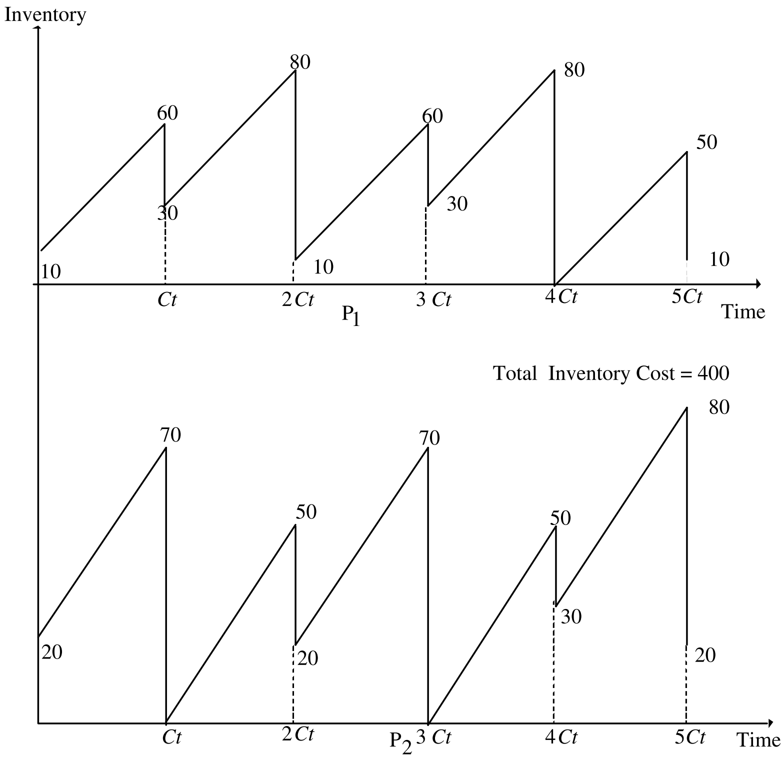


Fig. 3 Inventory level: Where manufacturer dominates: $\sigma^* = (50, 50, 50, 50, 50)$, $\nu(\sigma^*) = (1, 4, 5, 3, 2)$, $D_1 = (30, 40, 80, 70, 30)$, $D_2 = (70, 60, 20, 30, 70)$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_{ri} = 1, \quad r = 1, \dots, n, \tag{14}$$

$$\sum_{r=1}^{n} x_{ri} = 1, \quad i = 1, \dots, n, \tag{15}$$

$$I_{1,i} = I_{1,i-1} + z_{i1} - \sum_{r=1}^{n} d_{r1} x_{ri}, \quad i = 1, \dots, n, \tag{16}$$

$$I_{2,i} = I_{2,i-1} + z_{i2} - \sum_{r=1}^{n} d_{r2} x_{ri}, \quad i = 1, \dots, n, \tag{17}$$

$$I_{1,n} = I_{1,0}, \tag{18}$$

$$I_{2,n} = I_{2,0}, \tag{19}$$

$$z_{i1} + z_{i2} = C, \qquad i = 1, \dots, n, \tag{20}$$

$$I_{1,i} \geq 0, \quad i = 1, \dots, n, \tag{21}$$

$$I_{2,i} \geq 0, \quad i = 1, \dots, n, \tag{22}$$

$$x_{ri} \in \{0, 1\}, \quad i = 1, \dots, n; \ r = 1, \dots, n. \tag{23}$$

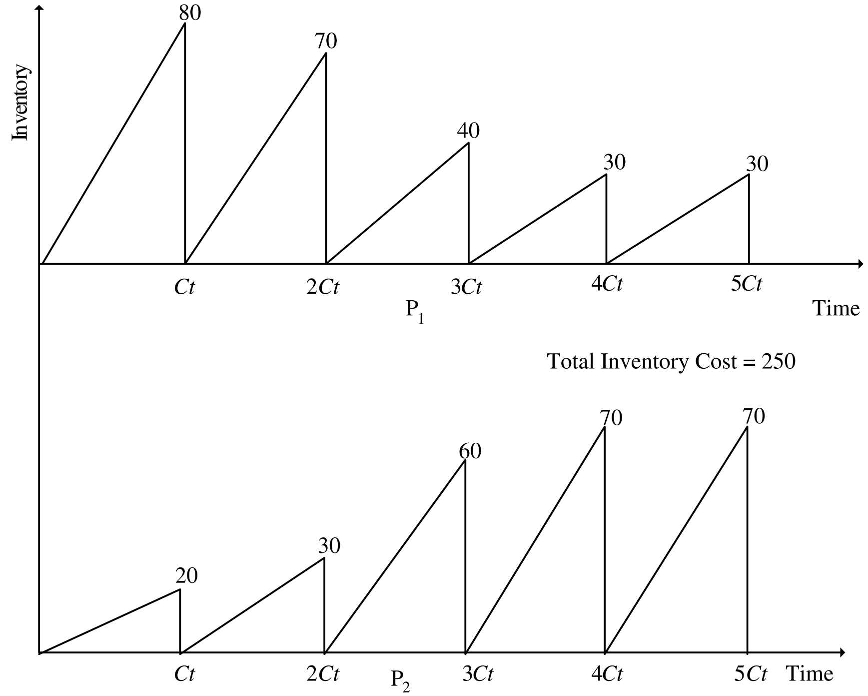


Fig. 4 Inventory levels: Where distributor dominates: $D_1 = (30, 40, 80, 70, 30)$, $D_2 = (70, 60, 20, 30, 70)$, $\sigma(\nu^*) = (80, 70, 40, 30, 30)$, $\nu^* = (3, 4, 2, 1, 5)$

Constraint (20) ensures the sum of P_1 and P_2 produced in a period equals the truck capacity.

For the example problem (in Sect. 2), the solution to the manufacturer's problem is presented in Fig. 4, where the production rate sequence, $\sigma(\nu^*) = (80, 70, 40, 30, 30)$ and the distribution sequence, $\nu^* = (3, 4, 2, 1, 5)$. $T(\sigma(\nu^*), \nu^*) = 250$ and $S(\sigma(\nu^*)) = 4 \times 25 = 100$. Note that in this solution, the constant term is $\frac{1}{2}(h_1 \sum_{i=1}^{n} d_{i1} + h_2 \sum_{i=1}^{n} d_{i2}) = 250$ and the variable term is $\sum_{i=0}^{n-1}(I_{1,i}h_1 + I_{2,i}h_2) = 0$.

We perform a computational study in Sect. 6 to determine the cost of the distributor's conflict. We compare the costs under the following two scenarios:

1. The manufacturer finds his optimal schedule, σ^*, first. Given σ^* the distributor determines his best distribution schedule $\nu(\sigma^*)$. The distributor's inventory holding cost in this scenario is $T(\sigma^*, \nu(\sigma^*))$.
2. Given the distributors optimal schedule ν^*, the manufacturer finds his best production rate schedule $\sigma(\nu^*)$. The distributor's inventory holding cost in this scenario is $T(\sigma(\nu^*), \nu^*)$.

3.2 Manufacturer's conflict

The manufacturer's conflict arises when the distributor dominates and decides his schedule first. The manufacturer is asked to change his production rate schedule according to the distribution schedule. The manufacturer's conflict, which is the percentage increase in the cost for the manufacturer over his unconstrained optimal schedule is:

$$[S(\sigma(\nu^*)) - S(\sigma^*)]100\,/S(\sigma^*), \tag{24}$$

where σ^* is the unconstrained optimal sequence for the manufacturer. It may be noted that $S(\sigma^*) = \mu$, $\sigma^* = (p_1, p_1, \dots, p_1)$, $p_1 = c\frac{\tau_1}{\tau_1+\tau_2}$. The distributor being the dominant partner sets his optimal distribution schedule ν^* and then forces the manufacturer to adopt the rate schedule $\sigma(\nu^*)$ that is optimal for the distributor. $S(\sigma(\nu^*))$ denotes the manufacturer's rate change cost in this scenario. The worst case scenario for the manufacturer is to make a rate change in every period that is $S(\sigma(\nu^*)) = n\mu$.

We determine $S(\sigma(\nu^*))$ as stated in Lemma 3, i.e., given ν^* obtain the minimum rate change cost, $S(\sigma(\nu^*))$. Recall that $\nu[i]$ denotes the ith retailer served in the distribution sequence ν.

Lemma 2 *For any distribution sequence ν, there exists a rate sequence $\sigma(\nu) = (p_{1j}, p_{2j}, \dots, p_{nj})$ $j = 1, 2$, where $p_{ij} = d_{\nu[i]j}$, $i = 1, 2, \dots, n$; $j = 1, 2$, that minimizes the total inventory cost for the distributor.*

Proof It can be easily verified that for any distribution sequence ν with $\sigma(\nu) = (p_{11}, p_{21}, \dots, p_{n1})$ and $p_{i1} = d_{\nu[i]1}$, $i = 1, 2, \dots, n$, the inventory cost for the distributor is $T(\sigma(\nu), \nu) = \frac{1}{2}\sum_{i=1}^{n}(d_{i1}h_1 + d_{i2}h_2)$. □

Lemma 3 *There exists a distribution sequence ν^* with $\sigma(\nu^*) = (d_{\nu^*[1]1}, d_{\nu^*[2]1}, \dots, d_{\nu^*[n]1})$ such that retailers' demands are served in the nonincreasing order $d_{\nu^*[1]1} \ge d_{\nu^*[2]1} \ge \cdots \ge d_{\nu^*[n]1}$ (or nondecreasing order $d_{\nu^*[1]1} \le d_{\nu^*[2]1} \le \cdots \le d_{\nu^*[n]1}$) which minimizes the number of rate changes for the manufacturer.*

Proof From Lemma 2, the inventory cost for the distributor, $T(\sigma(\nu_o), \nu_o) = \frac{1}{2}\sum_{i=1}^{n}(d_{i1}h_1 + d_{i2}h_2)$ remain constant for all distribution sequence ν_o with $\sigma(\nu_o) = (p_{11}, p_{21}, \dots, p_{n1})$ and $p_{i1} = d_{\nu_o[i]1}$, $i = 1, 2, \dots, n$. Let ν be a sequence in which demands are not served in nonincreasing order (or nondecreasing order) as in ν^*. In ν if there are adjacent equal demands then we call them a block otherwise a single element will form a block. If there exists two adjacent demands $d_{\nu[j]1}$ and $d_{\nu[j+1]1}$ which are respectively the last and first elements of block k and $k+1$, $k < n$, such that $d_{\nu[j]1} < d_{\nu[j+1]1}$ we can interchange the positions of the two blocks and obtain a new sequence ν'. It is evident that $S(\sigma(\nu)) \ge S(\sigma(\nu'))$. A finite number of such adjacent pairwise interchanges of blocks in ν will result in ν^*. Each such interchange will not result in the increase in rate change cost proving that $S(\sigma(\nu^*))$ is optimal. In any sequence ν' or ν^*, the distributor will be incurring the same cost. □

Theorem 2 *Suppose that the retailers are ordered so that $d_{11} \ge d_{21} \ge \cdots \ge d_{n1}$ (or $d_{11} \le d_{21} \le \cdots \le d_{n1}$). The distributor's schedule is $\nu^* = (1, 2, \dots, n)$. The Manufacturer's problem is solved by a production rate sequence $\sigma(\nu^*) = (p_{11}, p_{21}, \dots, p_{n1})$, where $p_{i1} = d_{i1}$, $i = 1, 2, \dots, n$, which minimizes his total rate change cost.*

Proof Follows from Lemma 2 and Lemma 3. □

For the example problem in Sect. 2, $\nu^* = (3, 4, 2, 5, 1)$ or $(1, 5, 2, 4, 3)$. In either of these schedules the manufacturer will make 4 production rate changes. The total inventory holding cost in both these sequences will be 250 and the manufacturer will incur a cost of 100 for the four rate changes, assuming $\mu = 25$. The inventory graph is shown in Fig. 4.

4 Computational complexity

We have seen in the previous section how the manufacturer and the distributor each optimize their individual problems. We now discuss the complexity of their scheduling problems if they do not have negotiating power within the supply chain.

4.1 Manufacturer dominates

For the two-product problem, the optimal solution for the manufacturer is to use only one production rate per distribution cycle, i.e, $\sigma^* = (p_1, p_1, \ldots, p_1)$, where $p_1 = C\frac{\tau_1}{(\tau_1+\tau_2)}$ and $p_2 = C - p_1$. Given this production rate schedule, the distributor must obtain a sequence that minimizes, over all possible distribution sequences, his cost. This distribution sequence is then repeated until the end of the planning horizon. For the example problem (Sect. 2), Fig. 3 shows the inventory graph of one of the feasible $n!$ sequences of the distributor.

Let $\nu(\sigma^*) = (\nu[1], \nu[2], \ldots, \nu[n])$ be the delivery sequence. We now explain how the inventory status of product P_j, $j = 1, 2$, can be computed over the cycle time nCt. Let the inventory holding cost from period 1 to period n for product P_j be denoted T'_j, $j = 1, 2$. Thus, $I'(\sigma^*, \nu(\sigma^*)) = T'_1 + T'_2$, where $I'(\sigma^*, \nu(\sigma^*))$ is the variable term in $T(\sigma^*, \nu(\sigma^*))$. We need the following preliminary result.

Lemma 4 *Given production rate sequence, $\sigma^* = (p_1, p_1, \ldots, p_1)$, the distributor's inventory holding cost can be expressed as follows*:

$$I'(\sigma^*, \nu(\sigma^*)) = nI_{1,0}h_1 + nI_{2,0}h_2 + \frac{n(n+1)}{2}(p_1 + p_2)(h_1 + h_2) - \sum_{i=1}^{n}[(n+1-i)h_1 d_{\nu[i]1} + (n+1-i)h_2 d_{\nu[i]2}].$$

Proof Figure 2 shows the inventory of both products during the distribution cycle. Initially, the inventory of product P_1 is $I_{1,0}$. At time Ct, the total inventory of product P_1 is $I_{1,0} + p_1 - d_{\nu[1]1}$, as retailer $R_{\nu[1]}$ is served in this period, and the inventory cost for this period is $h_1(I_{1,0} + p_1 - d_{\nu[1]1})$. At time $2Ct$, the total inventory of product P_1 is $I_{1,0} + 2p_1 - d_{\nu[1]1} - d_{\nu[2]1}$, as retailer $R_{\nu[2]}$ is served, and the inventory cost for this period is $h_1(I_{1,0} + 2p_1 - d_{\nu[1]1} - d_{\nu[2]1})$, and so on. Finally, at time nCt, when a delivery has been made to retailer $R_{\nu[n]}$, the inventory cost of product for this period P_1 is $h_1(I_{1,0} + np_1 - \sum_{i=1}^{n} d_{\nu[i]1})$. We thus have

$$T'_1 = h_1\left[nI_{1,0} + \frac{n(n+1)}{2}p_1 - \sum_{i=1}^{n}(n+1-i)d_{\nu[i]1}\right]. \tag{25}$$

Similarly, the inventory holding cost for period 1 to period n for product P_2 is

$$T'_2 = h_2\left[nI_{2,0} + \frac{n(n+1)}{2}p_2 - \sum_{i=1}^{n}(n+1-i)d_{\nu[i]2}\right]. \tag{26}$$

The result then follows from (25) and (26) as $I'(\sigma^*, \nu(\sigma^*)) = T'_1 + T'_2$. □

The total inventory holding cost savings if retailer $R_{\nu[1]}$ is served in the first position is $a_{1,\nu[1]} = nh_1 d_{\nu[1]1} + nh_2 d_{\nu[1]2}$. In general, we can compute $a_{r,\nu[i]}$, i.e., the total savings if retailer $R_{\nu[i]}$ is served in the rth position. Let $x_{ri} = 1$ if retailer $R_{\nu[i]}$ is served in the rth position; 0 otherwise. From Lemma 4, the distributor's problem can be modeled as the following assignment problem with side constraints. Note that $\frac{n(n+1)}{2}(p_1 + p_2)(h_1 + h_2)$ is a constant so minimizing $I'(\sigma^*, \nu(\sigma^*))$ reduces to the following mixed integer program.

$$\text{Minimize } nI_{1,0}h_1 + nI_{2,0}h_2 - \sum_{r=1}^{n}\sum_{i=1}^{n} a_{ri}x_{ri} \tag{27}$$

$$\text{s.t.} \quad \sum_{r=1}^{n} x_{ri} = 1, \quad i = 1, \dots, n, \tag{28}$$

$$\sum_{i=1}^{n} x_{ri} = 1, \quad r = 1, \dots, n, \tag{29}$$

$$I_{1,r-1} + p_1 - \sum_{i=1}^{n} d_{1i}x_{ri} = I_{1,r}, \quad r = 1, \dots, n, \tag{30}$$

$$I_{1,n} = I_{1,0}, \tag{31}$$

$$I_{2,r-1} + p_2 - \sum_{i=1}^{n} d_{2i}x_{ri} = I_{2,r}, \quad r = 1, \dots, n, \tag{32}$$

$$I_{2,n} = I_{2,0}, \tag{33}$$

$$I_{i,r} \geq 0, \quad i = 1, 2;\ r = 0, \dots, n, \tag{34}$$

$$x_{ri} \in \{0, 1\}, \quad r = 1, \dots, n;\ i = 1, \dots, n. \tag{35}$$

For this case in which production rate sequence is $\sigma = (p_1, p_1, \dots, p_1)$, where $p_1 = C\frac{\tau_1}{(\tau_1+\tau_2)}$ and $p_2 = C - p_1$, we have the following result for the distributor's problem.

Theorem 3 *For the production rate sequence $\sigma = (p_1, p_1, \dots, p_1)$, where $p_1 = C\frac{\tau_1}{(\tau_1+\tau_2)}$, finding an optimal sequence for the distributor is strongly NP-hard.*

Proof It can be shown that 3-Partition (3P) reduces to our problem.

3-Partition (Garey and Johnson 1979) *Given a set of positive integers $A = \{a_1, a_2, \dots, a_{3s}\}$, s and B with $\sum_{i=1}^{3s} a_i = sB$ and $\frac{B}{4} < a_i < \frac{B}{2}$, $i = 1, 2, \dots, 3s$, does there exist a three element partition in A, $\Gamma = \{\Gamma_1, \Gamma_2, \dots, \Gamma_s\}$ such that $\sum_{a_j \in \Gamma_i} a_j = B$, $1 \leq i \leq s$?*

We create an instance of the distributors problem when manufacturer dominates from the 3P problem. This involves two types of retailers R_{a_i} and R_{b_i} where 3P parameters a_i and B are part of retailer R_{a_i} and R_{b_i}'s orders respectively. We include a large constant L as part of the order size. The manufacturer produces a total of $2M$ units of both products (P_1 and P_2) every period. M is the average demand per period for each product and $M = 3(L + B)$.

We restate distributors problem as a decision problem:

Decision problem *Given a set of retailers with a given set of demands for products P_1 and P_2, does there exist a sequence $\nu^\star$, such that the total inventory cost for the sequence, $T(\nu^\star)$, is less than or equal to D?*

Define the above scheduling problem with $n = 4s$ retailers, with their demand for the product P_1 in the period i is as given below

$$\begin{aligned} R^1_{a_i} &= M - L - a_i, \quad && 1 \le i \le 3s, \\ R^1_{b_i} &= M + 3L + B, \quad && 1 \le i \le s, \end{aligned}$$

and that for product P_2 is

$$\begin{aligned} R^2_{a_i} &= M + L + a_i, \quad && 1 \le i \le 3s, \\ R^2_{b_i} &= M - 3L - B, \quad && 1 \le i \le s. \end{aligned}$$

The carrying cost associated with product P_1 and P_2 are h_1 and h_2 respectively, where $h_1 = h_2 = 1$. $D = 4sM + 12sL + 4sB$, where $L = 10B$.

At the end of each period the distributor will ship a combined total of $2M$ units of P_1 and P_2 from the manufacturer's facility. The total quantity shipped in a period is equal to his vehicle capacity C, where $C = 2M$.

The decision problem is clearly in class NP. One can easily verify that the construction of the decision problem from an instance of the 3P is polynomially bounded. Thus, we need to show that there exists a distributor's sequence $\nu^\star$ such that $T(\nu^\star) \le D$ if and only if there exists a solution to the 3P problem.

If part: Suppose there exists a solution to the 3P problem. Let the solution be $a_{3i-2} + a_{3i-1} + a_{3i} = B$, $1 \le i \le s$. Consider a distributor's sequence $\nu = (R_{a_1}, R_{a_2}, R_{a_3}, R_{b_1}, \dots, R_{a_{3i-2}}, R_{a_{3i-1}}, R_{a_{3i}}, R_{b_i}, \dots, R_{a_{3s-2}}, R_{a_{3s-1}}, R_{a_{3s}}, R_{b_s})$; now we show that the inventory for this sequence, $T(\nu) = D$. Figure 5 shows the inventory status of P_1 and P_2 for all $4s$ periods. From Fig. 5, it can be observed that $T(\nu) = 4sM + 12sL + 4sB$. Hence, the proof for the If part.

Only If part: Suppose there exists a sequence $\nu^\star$ such that $T(\nu^\star) \le D$. We argue that the schedule $\nu^\star$ can only take the form, $\nu^\star = (R_{a_1}, R_{a_2}, R_{a_3}, R_{b_1}, \dots, R_{a_{3s-2}}, R_{a_{3s-1}}, R_{a_{3s}}, R_{b_s})$.

Note that for any sequence ν, $T(\nu)$ can be written as $T(\nu) = 4sM + I'(\nu)$, here $I'(\nu)$ is the variable part of the inventory cost measured using inventory levels at the end of each period as discussed in Sect. 3. Note that in Fig. 5, $I'(\nu) = 12sL + 4sB$.

$\nu^\star$ can be constructed from the concatenation of s four-retailer sub-sequences $\nu^\star_P(i)$, $i = 1, \dots, s$, i.e., $\nu^\star = (\nu^\star_P(1), \nu^\star_P(2), \dots, \nu^\star_P(s))$. Because of the cyclic nature of $\nu^\star$, without any loss of generality we can fix the first retailer served by the distributor in $\nu^\star_P(1)$ as R_a (or R_b) type. First, we find all possible four-retailer sub-sequences $\nu_P(1)$ beginning with R_a and R_b type retailer and the corresponding inventory holding cost $I'(\nu_P(1))$ (see Table 1).

The following claims construct the sequence $\nu^\star$

Claim 1 $(R_{a_1}, R_{a_2}, R_{a_3}, R_{b_1})$, $(R_{a_1}, R_{b_1}, R_{a_2}, R_{a_3})$ *and* $(R_{a_1}, R_{a_2}, R_{b_1}, R_{a_3})$ *are the partial sequences which have the lowest $I'(\nu_P(1))$ values after fixing R_{a_1} as the first retailer served by the distributor.*

Claim 1 is established as follows: We evaluated the following eight partial sequences that begins with R_{a_1}:

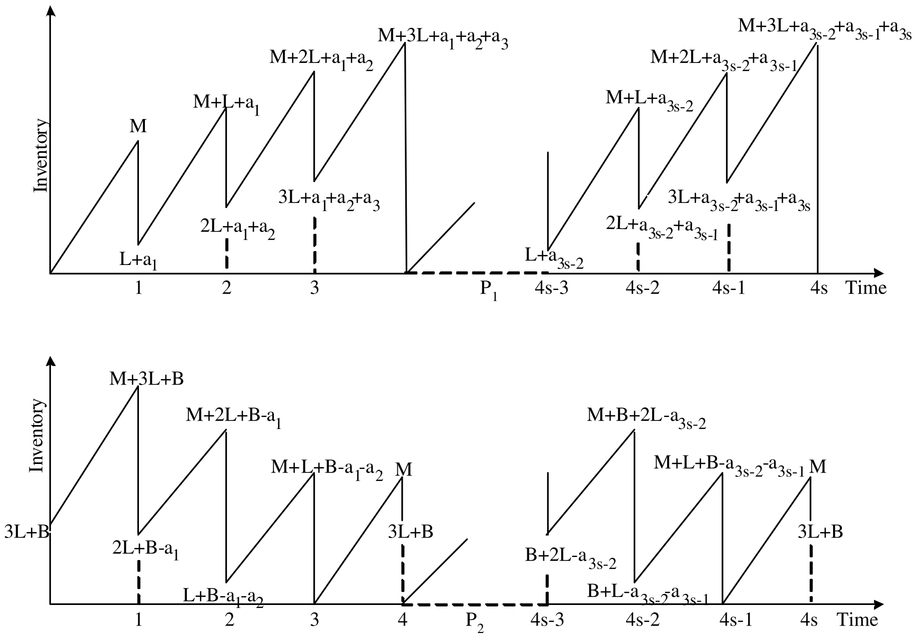


Fig. 5 $\nu = (R_{a_1}, R_{a_2}, R_{a_3}, R_{b_1}, \dots, R_{a_{3s-2}}, R_{a_{3s-1}}, R_{a_{3s}}, R_{b_s})$, $T(\nu) = D = 4sM + 12sL + 4sB$

Table 1 Table of subsequences

Seq beg. with	$\nu_P(1)$	$I'(\nu_P(1))$	Figure
R_{a_i}	$(R_{a_1}, R_{a_2}, R_{a_3}, R_{a_4})$	$16L + 4a_1 + 4a_2 + 4a_3$	Figure 7(a)
	$(R_{a_1}, R_{a_2}, R_{a_3}, R_{b_1})$	$12L + 4a_1 + 4a_2 + 4a_3$	Figure 7(b)
	$(R_{a_1}, R_{a_2}, R_{b_1}, R_{a_3})$	$12L + 4B$	Figure 8(a)
	$(R_{a_1}, R_{b_1}, R_{a_2}, R_{a_3})$	$12L + 4B$	Figure 8(b)
	$(R_{a_1}, R_{b_1}, R_{a_2}, R_{b_2})$	$20L + 8B - 4a_2$	Figure 9(a)
	$(R_{a_1}, R_{a_2}, R_{b_1}, R_{b_2})$	$24L + 8B$	Figure 9(b)
	$(R_{a_1}, R_{b_1}, R_{b_2}, R_{b_3})$	$36L + 12B$	Figure 10(a)
	$(R_{a_1}, R_{b_1}, R_{b_2}, R_{a_2})$	$24L + 8B$	Figure 10(b)
R_{b_i}	$(R_{b_1}, R_{b_2}, R_{b_3}, R_{b_4})$	$48L + 16B$	Figure 11(a)
	$(R_{b_1}, R_{b_2}, R_{b_3}, R_{a_1})$	$36L + 13B$	Figure 11(b)
	$(R_{b_1}, R_{b_2}, R_{a_1}, R_{b_3})$	$32L + 12B - 4a_1$	Figure 12(a)
	$(R_{b_1}, R_{a_1}, R_{b_2}, R_{b_3})$	$32L + 12B - 4a_1$	Figure 12(b)
	$(R_{b_1}, R_{a_1}, R_{a_2}, R_{b_2})$	$16L + 8B - 4a_1 - 4a_2$	Figure 13(a)
	$(R_{b_1}, R_{b_2}, R_{a_1}, R_{a_2})$	$24L + 8B$	Figure 13(b)
	$(R_{b_1}, R_{a_1}, R_{b_2}, R_{a_2})$	$20L + 8B - 4a_1 - 2a_2$	Figure 14(a)
	$(R_{b_1}, R_{a_1}, R_{a_2}, R_{a_3})$	$12L + 4B$	Figure 14(b)

$$(R_{a_1}, R_{a_2}, R_{a_3}, R_{a_4}), \quad (R_{a_1}, R_{a_2}, R_{a_3}, R_{b_1}), \quad (R_{a_1}, R_{a_2}, R_{b_1}, R_{a_3}),$$
$$(R_{a_1}, R_{b_1}, R_{a_2}, R_{a_3}), \quad (R_{a_1}, R_{b_1}, R_{a_2}, R_{b_2}), \quad (R_{a_1}, R_{a_2}, R_{b_1}, R_{b_2}),$$
$$(R_{a_1}, R_{b_1}, R_{b_2}, R_{b_3}), \quad (R_{a_1}, R_{b_1}, R_{b_2}, R_{a_2}),$$

refer Table 1 and Figs. 7–10 in Appendix. It can be seen that the partial sequences $(R_{a_1}, R_{a_2}, R_{a_3}, R_{b_1})$, $(R_{a_1}, R_{b_1}, R_{a_2}, R_{a_3})$ and $(R_{a_1}, R_{a_2}, R_{b_1}, R_{a_3})$ have the lowest $I'(\nu_P(1))$ values, $12L + g(a, B)$. Here $g(a, B)$ is called the garbage function which collects all pertinent 3P parameters. $g(a, B)$ is an integer valued function and $g(a, B) < L$. Since $g(a, B)$ are very small as compared to L, the actual values of $g(a, B)$ are not important for the purpose of comparing subsequences. Figures 7–10 show inventory graph for P_1 and P_2, $I'(\nu_P(1))$ is the total inventory carrying cost for product P_1 and P_2 by considering inventory levels at the end of periods 1, 2, 3 and 4. Note that the initial inventory cost at the end of period 0 is not accounted in $I'(\nu_P(1))$. The partial sequence $\nu_P(s)$ includes the initial inventory cost in $I'(\nu_P(s))$ as the initial inventory is the same as the final inventory at the end of period $4s$.

Claim 2 $(R_{b_1}, R_{a_1}, R_{a_2}, R_{a_3})$ *is the partial sequences with minimum* $I'(\nu_P(1))$ *value after fixing* R_{b_1} *as the first retailer served.*

Claim 2 is established as follows: We evaluated the following eight partial sequences, $(R_{b_1}, R_{b_2}, R_{b_3}, R_{b_4})$, $(R_{b_1}, R_{b_2}, R_{b_3}, R_{a_1})$, $(R_{b_1}, R_{b_2}, R_{a_1}, R_{b_3})$, $(R_{b_1}, R_{a_1}, R_{b_2}, R_{b_3})$, $(R_{b_1}, R_{a_1}, R_{a_2}, R_{b_2})$, $(R_{b_1}, R_{b_2}, R_{a_1}, R_{a_2})$, $(R_{b_1}, R_{a_1}, R_{b_2}, R_{a_2})$, $(R_{b_1}, R_{a_1}, R_{a_2}, R_{a_3})$. Note that $(R_{b_1}, R_{a_1}, R_{a_2}, R_{a_3})$ has the minimum $I'(\nu_P(1))$ value, $12L + g(a, B)$ (Table 1). Hence the proof.

Table 1 shows the inventory cost of all possible 4-retailer subsequences. As $I'(\nu^\star) \le 12sL + 4sB$, we may construct $\nu^\star$ with s subsequences having the inventory cost of $12L$ (we ignore the values of the garbage functions as they are very small compared to L). Thus, $\nu^\star$ may take one of the following form:

$\hat{\nu}_1$: Repetitive s subsequences of $(R_{a_i}, R_{a_j}, R_{a_k}, R_{b_i})$. That is

$$\hat{\nu}_1 = (R_{a_1}, R_{a_2}, R_{a_3}, R_{b_1}), (R_{a_4}, R_{a_5}, R_{a_6}, R_{b_2}), \ldots, (R_{a_{3s-2}}, R_{a_{3s-1}}, R_{a_{3s}}, R_{b_s}).$$

$\hat{\nu}_2$: Repetitive s subsequences of $(R_{b_i}, R_{a_i}, R_{a_j}, R_{a_k})$.
$\hat{\nu}_3$: Repetitive s subsequences of $(R_{a_i}, R_{a_j}, R_{b_i}, R_{a_k})$.
$\hat{\nu}_4$: Repetitive s subsequences of $(R_{a_i}, R_{b_i}, R_{a_j}, R_{a_k})$.
$\hat{\nu}_5$: Composes of combination of s subsequences of types: $(R_{a_i}, R_{a_j}, R_{a_k}, R_{b_i})$, $(R_{b_i}, R_{a_i}, R_{a_j}, R_{a_k})$, $(R_{a_i}, R_{b_i}, R_{a_j}, R_{a_k})$ and $(R_{a_i}, R_{a_j}, R_{b_i}, R_{a_k})$.

Claim 3 *Sequences* $\hat{\nu}_1$, $\hat{\nu}_2$, $\hat{\nu}_3$ *and* $\hat{\nu}_4$ *are identical.*

Claim 3 is established as follows: Due to the cyclic nature of $\nu^\star$, s repetitive sequences of $(R_{a_i}, R_{a_j}, R_{a_k}, R_{b_i})$ is identical to s repetitive sequences of $(R_{b_i}, R_{a_i}, R_{a_j}, R_{a_k})$ which in turn is identical to s repetitive sequences of $(R_{a_i}, R_{a_j}, R_{b_i}, R_{a_k})$ (or $(R_{a_i}, R_{b_i}, R_{a_j}, R_{a_k})$).

Claim 4 *Sequence* $\hat{\nu}_5$ *cannot be* $\nu^\star$.

Claim 4 is established as follows: Due to a mismatch of initial and final inventory levels of the subsequences, $(R_{a_i}, R_{a_j}, R_{a_k}, R_{b_i})$, $(R_{a_i}, R_{b_i}, R_{a_j}, R_{a_k})$, $(R_{b_i}, R_{a_i}, R_{a_j}, R_{a_k})$ and

$(R_{a_i}, R_{a_j}, R_{b_i}, R_{a_k})$, $I'(\hat{\nu}_5)$ cannot be less than or equal to $12sL + 4sB$. The quantity of the mismatch between any of two subsequences is at least L which is a large number and cause $\hat{\nu}_5$ to have the inventory cost, $I'(\hat{\nu}_5)$, more than $12sL + 4sB$.

As the consequence of the above claims, the form of $\nu^\star$ is s repetitive subsequences of $(R_{a_i}, R_{a_j}, R_{a_k}, R_{b_i})$ each having the inventory cost of $12L + g(a,b)$. That is, $\nu^\star_P(i) = (R_{a_{3i-2}}, R_{a_{3i-1}}, R_{a_{3i}}, R_{b_i})$ and

$$\nu^\star = (\nu^\star_P(1), \nu^\star_P(2), \dots, \nu^\star_P(s)) = (R_{a_1}, R_{a_2}, R_{a_3}, R_{b_1}, R_{a_4}, R_{a_5}, R_{a_6}, R_{b_2}, \dots,$$
$$R_{a_{3s-2}}, R_{a_{3s-1}}, R_{a_{3s}}, R_{b_s}).$$

Let Γ_i is a set of three elements, a_{3i-2}, a_{3i-1}, a_{3i}, corresponding to subsequence $\nu^\star_P(i)$, $i = 1, \dots, s$.

If $a_{3i-2} + a_{3i-1} + a_{3i} < B$ then there will be an extra inventory of $(B - a_{3i-2} - a_{3i-1} - a_{3i})$ that has to be carried from one subsequence to the subsequent subsequence. Consequently, the inventory cost, $I'(\nu^\star)$ will be more than $12sL + 4sB$. Refer Fig. 15 for a 8-period problem where $s = 2$ and $(a_1 + a_2 + a_3) < (a_4 + a_5 + a_6)$. Note that an extra inventory of $(B - a_1 - a_2 - a_3)$ is carried from one subsequence to the subsequent subsequence.

If $a_{3i-2} + a_{3i-1} + a_{3i} > B$ then there will be an extra inventory of $(a_{3i-2} + a_{3i-1} + a_{3i} - B)$ that has to be carried from one subsequence to the subsequent subsequence. Consequently, the inventory cost, $I'(\nu^\star)$ will be more than $12sL + 4sB$. In Fig. 15, an extra inventory of $(a_4 + a_5 + a_6 - B)$ is carried from one subsequence to the subsequent subsequence.

Since $\sum_{i=1}^{3s} a_i = sB$ and $I'(\nu^\star) \le 12sL + 4sB$ we conclude that $a_{3i-2} + a_{3i-1} + a_{3i} = B$, $i = 1, 2, \dots, s$. This implies A can be partitioned into s disjoint subsets $\Gamma_1, \Gamma_2, \dots, \Gamma_s$. Hence 3-Partition has a solution and the proof for Theorem 3. Figure 5 shows the inventory graph for $\nu^\star = (R_{a_1}, R_{a_2}, R_{a_3}, R_{b_1}, R_{a_4}, R_{a_5}, R_{a_6}, R_{b_2}, \dots, R_{a_{3s-2}}, R_{a_{3s-1}}, R_{a_{3s}}, R_{b_s})$. □

4.2 Distributor dominates

The manufacturer produces according to a schedule $\sigma(\nu^*)$ preferred by the distributor. That is, the distributor chooses $\sigma(\nu^*)$ for the manufacturer such that ν^* is optimal for the distributor, i.e., the distributor's inventory holding cost, $T(\sigma(\nu^*), \nu^*)$ is minimum. Several combinations of sequences, $(\sigma(\nu^*), \nu^*)$, provide the minimum inventory cost, $T(\sigma(\nu^*), \nu^*)$. The distributor is indifferent to any of the sequence combination. Thus, the manufacturer's problem is to find the sequence combination, $(\sigma(\nu^*), \nu^*)$ that minimizes his rate change cost.

Theorem 4 *When the distributor dominates, the manufacturer's problem of finding the sequence combination, $(\sigma(\nu^*), \nu^*)$ that minimizes his rate change cost is polynomially solvable in $O(n \log n)$ computational steps.*

Proof Suppose that the retailers are ordered so that $d_{11} \ge d_{21} \ge \cdots \ge d_{n1}$ (or $d_{11} \le d_{21} \le \cdots \le d_{n1}$). The distributor's schedule is $\nu^* = (1, 2, \dots, n)$. The manufacturer's problem is solved by a production rate sequence $\sigma(\nu^*) = (p_{11}, p_{21}, \dots, p_{n1})$, where $p_{i1} = d_{i1}$, $i = 1, 2, \dots, n$, which minimizes his total rate change cost. It follows from the Theorem 2 that arranging the retailers in the increasing (or decreasing order) of demand size and setting $\sigma(\nu^*) = (p_{11}, p_{21}, \dots, p_{n1})$ with $p_{i1} = d_{i1}$, $i = 1, 2, \dots, n$, will yield minimum rate change cost for the manufacturer. Note that the sequence combination $(\sigma(\nu^*), \nu^*)$ yields minimum inventory cost for the distributor. The time complexity of sort algorithm is $O(n \log n)$ which is polynomial for a fixed n. □

5 Benefit of cooperation

The total system cost can be reduced if the manufacturer and the distributor cooperate and make joint decisions. In this section, we evaluate and compare the total supply chain cost for the joint decision with the costs of independent decisions. Methods to implement cooperation are also discussed in this section.

If the manufacturer and the distributor cooperate, then they need to find the manufacturer's schedule, $\hat{\sigma}$ and the distributor's schedule, $\nu(\hat{\sigma})$ that jointly minimizes the total system cost, $S(\hat{\sigma}) + T(\hat{\sigma}, \nu(\hat{\sigma}))$.

Theorem 5 *The system problem of finding the sequence combination* $(\hat{\sigma}, \nu(\hat{\sigma}))$ *to minimize the total system cost*, $S(\hat{\sigma}) + T(\hat{\sigma}, \nu(\hat{\sigma}))$ *is strongly NP-hard.*

Proof Consider the problem instance with $\mu = \infty$ (very large rate change cost) in which the system must have only one production rate throughout the distribution cycle. Thus, in an optimal solution, we must have $\hat{\sigma} = (p_1, \dots, p_1)$ with $p_1 = C\frac{\tau_1}{\tau_1+\tau_2}$. Given $\hat{\sigma} = (p_1, \dots, p_1)$, we now need to find $\nu(\hat{\sigma})$ that minimizes $T(\hat{\sigma}, \nu(\hat{\sigma}))$, which is precisely the distributor's problem when manufacturer dominates. This problem is shown to be strongly NP-hard in Theorem 3. □

The system problem can be formulated as the following mixed integer program as shown below, where, we let $y_i = 1$ if the production rate is changed from period $i-1$ to period i, $y_i = 0$ otherwise. M is a large number. We set $M = 2C$.

$$\text{Minimize } \mu \sum_{i=1}^{n} y_i + \left[h_1 \sum_{i=1}^{n} I_{1,i} + h_2 \sum_{i=1}^{n} I_{2,i} \right]$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_{ri} = 1, \quad r = 1, \dots, n, \tag{36}$$

$$\sum_{r=1}^{n} x_{ri} = 1, \quad i = 1, \dots, n, \tag{37}$$

$$I_{1,i} = I_{1,i-1} + z_{i1} - \sum_{r=1}^{n} d_{r1} x_{ri}, \quad i = 1, \dots, n, \tag{38}$$

$$I_{2,i} = I_{2,i-1} + C - z_{i1} - \sum_{r=1}^{n} d_{r2} x_{ri}, \quad i = 1, \dots, n, \tag{39}$$

$$I_{1,0} = I_{1,n}, \tag{40}$$

$$I_{2,0} = I_{2,n}, \tag{41}$$

$$I_{1,i} \ge 0, \quad i = 1, \dots, n, \tag{42}$$

$$I_{2,i} \ge 0, \quad i = 1, \dots, n, \tag{43}$$

$$M y_i \ge z_{i,1} - z_{i-1,1}, \quad i = 1, \dots, n, \tag{44}$$

$$M y_i \ge z_{i-1,1} - z_{i,1}, \quad i = 1, \dots, n, \tag{45}$$

$$z_{01} = z_{n1}, \tag{46}$$

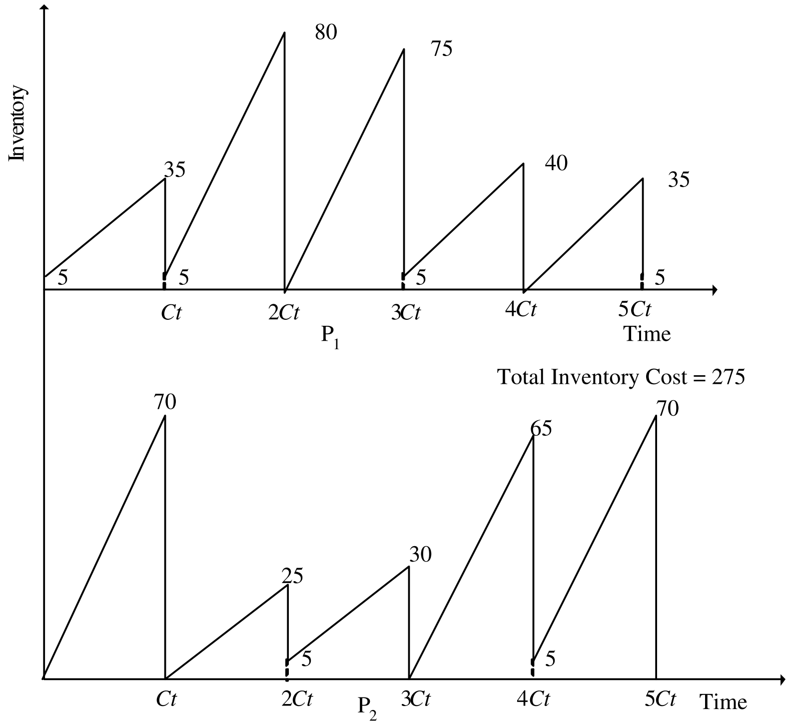


Fig. 6 Inventory level: For the case of cooperation: $\hat{\sigma} = (30, 75, 75, 35, 35)$, $\nu(\hat{\sigma}) = (1, 3, 4, 2, 5)$, $D_1 = (30, 40, 80, 70, 30)$, $D_2 = (70, 60, 20, 30, 70)$

$$y_i \in \{0, 1\}, \quad i = 1, \dots, n, \tag{47}$$

$$x_{ri} \in \{0, 1\}, \quad i = 1, \dots, n; \; r = 1, \dots, n. \tag{48}$$

Constraints (44) and (45) determines the required production rate changes.

For the example problem given in Sect. 2, the system problem is solved using the mixed integer program formulation given above. The solution is presented in Fig. 6. The rate sequence, $\hat{\sigma} = (30, 75, 75, 35, 35)$, the distributor's sequence, $\nu(\hat{\sigma}) = (1, 3, 4, 2, 5)$ and the inventory holding cost, $T(\hat{\sigma}, \nu(\hat{\sigma})) = 275$. Note that in the solution, the constant term, $\frac{n}{2}(p_1h_1 + p_2h_2) = 250$, the variable term, $\sum_{i=0}^{n-1}(I_{1,i}h_1 + I_{2,i}h_2) = 25$ and $S(\sigma^*) = 75$.

We perform a computational study in Sect. 7 to determine the relative cost savings to the supply chain that results from cooperation between the two parties. For each randomly generated problem, we evaluate costs under three different scenarios:

1. The manufacturer is the dominant partner and decides his best schedule σ^*, and the distributor determines his best schedule $\nu(\sigma^*)$ given the manufacturer's schedule σ^*. Cost of this scenario is given by $\Gamma^* = S(\sigma^*) + T(\sigma^*, \nu(\sigma^*))$.
2. The distributor is the dominant partner and the manufacturer finds his best schedule, $\sigma(\nu^*)$, given the distributors schedule, ν^*. Cost of this scenario is given by $\bar{\Gamma} = S(\sigma(\nu^*)) + T(\sigma(\nu^*), \nu^*)$.
3. The manufacturer and the distributor coordinate and determine schedules, $(\hat{\sigma}, \nu(\hat{\sigma}))$, that minimizes the overall system cost. We denote the cost of this scenario by $\hat{\Gamma} = S(\hat{\sigma}) + T(\hat{\sigma}, \nu(\hat{\sigma}))$.

Table 2 Supply chain costs for the example problem

Dominant partner	Manufacturing sequence	Distribution sequence	Manufacturer's cost	Distributor's cost	System cost
Manufacturer	$\sigma = (50, 50, 50, 50, 50)$	$\nu = (1, 4, 5, 3, 2)$	25	400	425
Distributor	$\sigma = (80, 70, 40, 30, 30)$	$\nu = (3, 4, 2, 1, 5)$	100	250	350
Cooperate	$\sigma = (30, 75, 75, 35, 35)$	$\nu = (1, 3, 4, 2, 5)$	75	275	350

Table 3 Supply chain conflicts for the example problem

Dominant partner	Manufacturing sequence	Distribution sequence	Manufacturer's conflict (%)	Distributor's conflict (%)
Manufacturer	$\sigma = (50, 50, 50, 50, 50)$	$\nu = (1, 4, 5, 3, 2)$	0	60
Distributor	$\sigma = (80, 70, 40, 30, 30)$	$\nu = (3, 4, 2, 1, 5)$	300	0
Cooperate	$\sigma = (30, 75, 75, 35, 35)$	$\nu = (1, 3, 4, 2, 5)$	200	10

The relative gain from using the cooperative schedule over the distributor's preferred schedule is computed as $(\bar{\Gamma} - \hat{\Gamma})/\bar{\Gamma}$, while that over the manufacturer's preferred schedule is given by $(\Gamma^* - \hat{\Gamma})/\Gamma^*$.

Table 2 shows the total system costs for different levels of cooperation for the example problem given in Sect. 2. It can be seen that the supply chain cost can be reduced if the two partners cooperate and make joint decisions. In the cooperative solution the dominant partner's cost increases whereas the cost of the other partner decreases. As shown in Table 2, if the manufacturer dominates, then his cost increases from \$25 to \$75 in the cooperative solution. On the other hand, the distributor's cost decreases from \$400 to \$275 in the cooperative solution. The system cost decreases from \$425 to \$350. Similar observations will be made if the distributor is the dominant partner. If the manufacturer is the dominant partner, then the manufacturer incurs an additional cost of \$50 to deviate from his optimal schedule whereas the distributor saves \$125 because of the manufacturer's cooperation. However, for cooperation to be effective and attractive to the partner whose cost increases because of this cooperation, the non-dominant partner must use some of his cost savings to compensate for the loss incurred by the dominant partner for changing his schedule. Nash (1953), states that such compensation needs to be a little more than the amount by which the dominant player's cost increases. In some of the recent papers Williamson (1975), Ouchi (1980), Corfman and Lehman (1993) and Lehman (2001) suggest that the surplus should be divided more equitably to ensure continued cooperation. In order to ensure that the surplus is shared in an equitable manner, both partners should be in a position to verify information related to the other partner's cost.

In this example, the distributor may give some compensation to the manufacturer (say \$55, which is more than the manufacturer's cost increase) to motivate him to deviate from his optimal sequence. This cooperation will decrease the manufacturer's cost from \$25 to \$20, the distributor's cost from \$400 to \$330 and the system cost from \$425 to \$350. Thus, both partners as well as the system benefit from the cooperation. Table 3 shows the amount of conflicts (calculated using (2) for the distributor and (24) for the manufacturer) for the example problem.

6 Computational study

In this section we solve all three problems optimally and demonstrate benefits of coordinated decision making. We use CPLEX 8.01 linear optimizer to solve the distributor's problem and the system problem. The manufacturer's problem, as we know from Theorem 4, can be solved optimally by a polynomial time algorithm. We find that the costs of conflict are significant.

6.1 Data set

Five different problem instances were generated for each chosen value of n. The demand values for the set D_j, $j = 1, 2$, were integer values generated randomly from $U[10, 95]$ distribution. The value of μ is set equal to \$25 and that of $C = 100$ for all our test problems. The values of n and combination of h_1 and h_2 were varied during experiments as discussed below.

6.2 Experimental results

The CPLEX 8.01 solves the distributor's problem of size up to $n = 30$ in reasonable amount of computing time, i.e., less than 3 hours. However, the solution time grows exponentially when n is above 30. The solution time, which varies between 0.11 second to 3 hours in an Intel Pentium Xeon Dual Processor 2.4 Ghz, depends on the problem size (n). For the system problem, the maximum problem size that CPLEX 8.01 could solve in less than 3 hours is $n = 9$. So we limited our experiments to problem sizes of up to $n = 9$.

Table 4 shows the total inventory costs for the three cases under study: the manufacturer dominates, the distributor dominates and the two partners cooperate. For all three problem scenarios the following combination of holding costs were used: ($h_1 = 1, h_2 = 1$), ($h_1 = 1.5, h_2 = 1$), ($h_1 = 2, h_2 = 1$). Table 4 shows the results for the first two of these combinations of holding costs for four different values of n, $n = \{9, 8, 7, 6\}$. All figures shown in the table are inventory holding costs in dollar values. Columns 3 and 7 show the inventory cost values when Distributor dominates. Inventory costs when Manufacturer dominates are shown in Columns 4 and 8, whereas those for the cooperation are shown in columns 5 and 9.

6.3 Conflict

Conflict measures the relative increase in cost of the non-dominating partner when the dominating partner imposes his optimal schedule on the non-dominating partner. To find the cost of conflict we use the results obtained from solving the individual problems optimally. The costs of conflict are significant, as discussed in this section.

Distributor's conflict

The distributor's conflict is the percentage increase in the distributor's inventory cost when the manufacturer adheres to his optimal schedule. Recall that the conflict is calculated as follows: $[T(\sigma^*, \nu(\sigma^*)) - T(\sigma(\nu^*), \nu^*)]100/T(\sigma(\nu^*), \nu^*)$. Table 5 shows the distributor's conflict in percentage for different combinations of the holding cost values and n. Each row in this table corresponds to an average of the five problems for each combination of n and (h_1 and h_2) presented in Table 4. The cost of conflict is significant, as can be seen from this table. The distributor incurs substantial increase in the inventory cost when he has to follow the manufacturer's optimal schedule. Hence, it will be in the best interest of the distributor to bring down his inventory cost by encouraging the manufacturer to move towards system optimal solution through a coordination mechanism.

Table 4 Total inventory cost for the three problems

	n	Dist. Dominates $T(\sigma(\nu^*), \nu^*)$	Mfg. Dominates $T(\sigma^*, \nu(\sigma^*))$	Cooperation $T(\hat{\sigma}, \nu(\hat{\sigma}))$	n	Dist. Dominates $T(\sigma(\nu^*), \nu^*)$	Mfg. Dominates $T(\sigma^*, \nu(\sigma^*))$	Cooperation $T(\hat{\sigma}, \nu(\hat{\sigma}))$
$h_1 = 1, h_2 = 1$	9	450.0	801.0	495.0	8	400.0	640.0	488.0
		450.0	819.0	486.0		400.0	672.0	424.0
		400.0	810.0	504.0		400.0	728.0	440.0
		450.0	747.0	486.0		400.0	632.0	464.0
		450.0	756.0	504.0		400.0	688.0	440.0
$h_1 = 1, h_2 = 1$	7	350.0	532.0	385.0	6	300.0	474.0	324.0
		350.0	546.0	392.0		300.0	474.0	324.0
		350.0	532.0	385.0		300.0	474.0	324.0
		350.0	560.0	378.0		300.0	426.0	324.0
		350.0	595.0	406.0		300.0	522.0	326.0
$h_1 = 1.5, h_2 = 1$	9	564.8	970.8	615.8	8	492.0	775.0	538.0
		569.3	997.3	610.3		516.0	826.0	544.5
		580.3	997.8	641.5		518.0	897.5	564.0
		542.3	887.8	584.3		490.0	758.5	509.0
		582.8	942.8	593.3		510.0	846.5	556.0
$h_1 = 1.5, h_2 = 1$	7	423.5	640.5	465.0	6	367.5	575.0	463.0
		444.5	676.0	491.0		382.5	588.5	493.0
		463.8	680.8	505.3		402.0	609.5	532.0
		427.0	673.5	459.5		358.5	509.0	446.0
		448.0	738.5	516.0		381.0	645.5	504.0

Table 5 Distributor's conflict (in %) for various h_1 and h_2 values

n	Dist. conflict (%) ($h_1 = 1, h_2 = 1$)	Dist. conflict (%) ($h_1 = 1.5, h_2 = 1$)	Dist. conflict (%) ($h_1 = 2, h_2 = 1$)
9	74.8	69.0	65.0
8	68.0	62.0	59.0
7	58.0	55.0	52.3
6	58.0	55.0	53.0

Table 6 Manufacturer's conflict (in %) for different n values

Periods, n	9	8	7	6
Conflict in %	780	680	560	500

Manufacturer's conflict

The manufacturer's conflict arises when the distributor dominates. The manufacturer is forced to deviate from his unconstrained optimal schedule, which is one rate change for the entire planning horizon. The manufacturers conflict is calculated as $[S(\sigma(\nu^*)) - S(\sigma^*)]100\ /S(\sigma^*)$. The values for the manufacturer's conflict are given in Table 6. The manufacturer's cost of conflict increases when the problem size grows larger. Each entry in Table 6 corresponds to the average of five test problems.

Note that the manufacturer's cost of conflict is also significant. Let's assume a rate change cost of \$25, which implies under this test scenario the manufacturer will incur cost anywhere (500% to 780%, from Table 6) between \$125 (25\$ $\times$ 500%) to \$195 (\$25 $\times$ 780%), depending on the number of periods, over his unconstrained optimal schedule cost of \$25. In order to reduce his cost in such a scenario, the manufacturer will negotiate with the distributor to establish a coordination mechanism.

7 Cooperation

In this section we show the benefit of cooperation between the manufacturer and the distributor. Instead of taking individual actions, they cooperate and take combined decisions. Table 7 shows the benefit of cooperation, measured in terms of percentage reduction in cost, for the distributor and the manufacturer when they move from the dominated status to the cooperative decision making.

7.1 Coordination mechanisms

Our computational study indicates that coordination can bring benefits to the supply chain members. We briefly explore the ways to implement coordination mechanisms in the two stage supply chain considered in this paper. The basic underlying fact is that there is a surplus in the system which is equivalent to $[\Gamma^* - \hat{\Gamma} = S_D]$ when manufacturer dominates and $[\bar{\Gamma} - \hat{\Gamma} = S_M]$ when distributor dominates. The surplus is defined as the savings for the non-dominant partner after covering the extra cost that the dominant partner incurs to move from an unconstrained optimal schedule to the coordinated schedule. If the manufacturer is the dominant partner and moves from an optimal to the coordinated schedule he will incur an extra cost equivalent to $[S(\hat{\sigma}) - S(\sigma^*)]$. The distributor's cost reduces by an amount $[T(\sigma^*, \nu(\sigma^*)) - T(\hat{\sigma}, \nu(\hat{\sigma}))]$. The surplus in this case is quantified as

Table 7 Percentage reduction in cost for the non-dominating member when there is cooperation

	Manufacturer dominates % reduction in cost for the distributor				Distributor dominates % reduction in cost for the manufacturer			
	$n=9$	$n=8$	$n=7$	$n=6$	$n=9$	$n=8$	$n=7$	$n=6$
$h_1=1, h_2=1$	37.0	33.0	30.0	31.0	50.0	56.4	51.5	50.0
$h_1=1.5, h_2=1$	36.0	34.0	28.0	30.0	45.5	46.2	51.5	50.0
$h_1=2, h_2=1$	35.0	33.0	28.0	29.0	45.5	46.2	48.5	50.0

Table 8 Surplus in the system

	Distributor's Surplus in \$ $(\Gamma^* - \hat{\Gamma})$				Manufacturer's Surplus in \$ $(\bar{\Gamma} - \hat{\Gamma})$			
	$n=9$	$n=8$	$n=7$	$n=6$	$n=9$	$n=8$	$n=7$	$n=6$
$h_1=1, h_2=1$	181.6	135.8	108.8	74.6	65.0	58.8	45.8	50.6
$h_1=1.5, h_2=1$	255.2	198.4	139.4	47.9	58.9	40.0	38.9	44.0
$h_1=2, h_2=1$	304.8	236.8	170.2	154.8	53.8	47.8	32.2	39.4

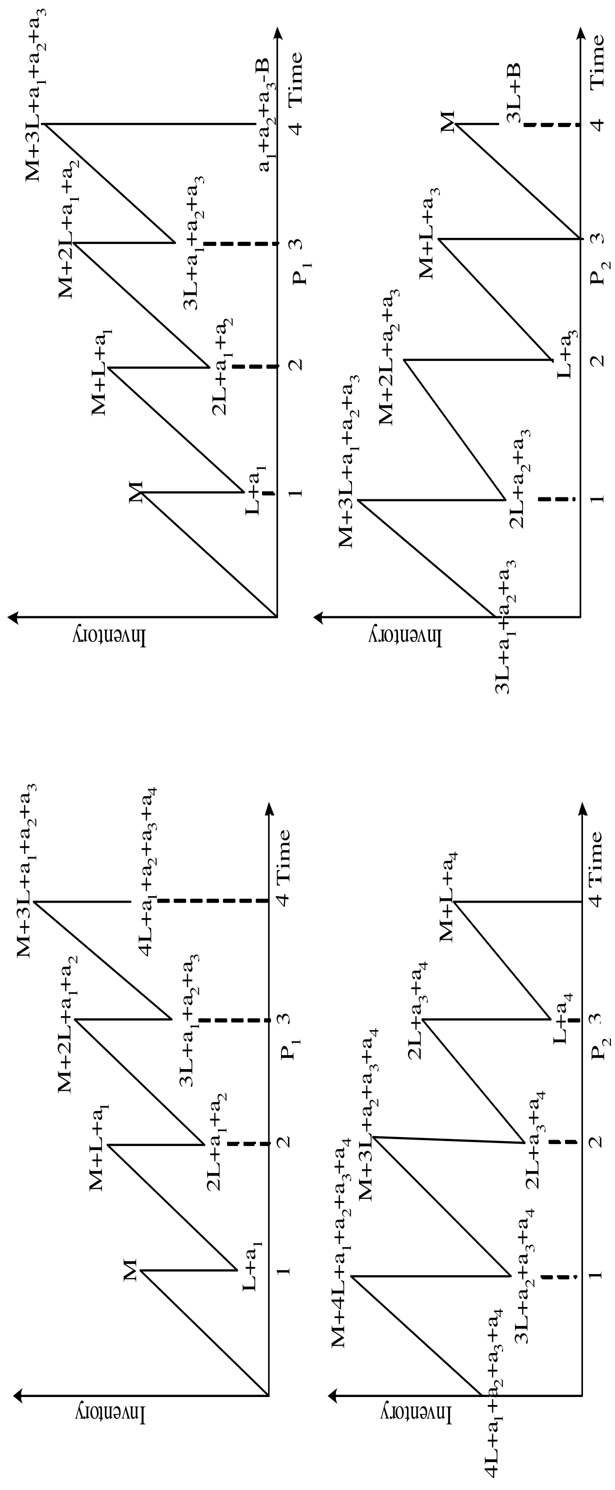
$[T(\sigma^*, \nu(\sigma^*)) - T(\hat{\sigma}, \nu(\hat{\sigma}))] - [S(\hat{\sigma}) - S(\sigma^*)] = S_D$. On the other hand, if the distributor is the dominating partner and moves from an optimal schedule to the coordinated schedule then manufacturer's surplus is given by $[S(\sigma(\nu^*) - S(\hat{\sigma})] - [T(\hat{\sigma}, \nu(\hat{\sigma})) - T(\sigma(\nu^*), \nu^*)] = S_M$. In most of the cases manufacturer is the dominant partner, but the distributor could be a dominant partner since timely delivery of finished goods is very important in a JIT environment.

As discussed in Sect. 5, the compensation for increased cost of moving from an optimal schedule to the coordinated schedule alone may not motivate the dominating partner to move to a coordinated schedule. A share in the surplus will make the move more attractive and desirable. Table 8 shows the surplus in the system, as explained in the previous paragraph, for various problems studied in this paper. Since the surplus is positive for all our test cases, we may conclude that it is possible for both parties to persuade the other to agree on a coordination mechanism by way of sharing the surplus.

The savings and gains for the non-dominant and dominant partner would be $S_D - \varepsilon$ (or $S_M - \varepsilon$, whoever dominates) and ε respectively, where ϵ is $\alpha(\Gamma^* - \hat{\Gamma})$ if manufacturer dominates and $\beta(\bar{\Gamma} - \hat{\Gamma})$ if distributor dominates where, $0 < \alpha, \beta < 1$. The value of α and β will depend on the bargaining power of the distributor and the manufacturer, respectively. For the coordination mechanism to work smoothly there should be trust between the two partners and sufficient information flow to verify rate change costs and inventory holding costs.

8 Conclusions

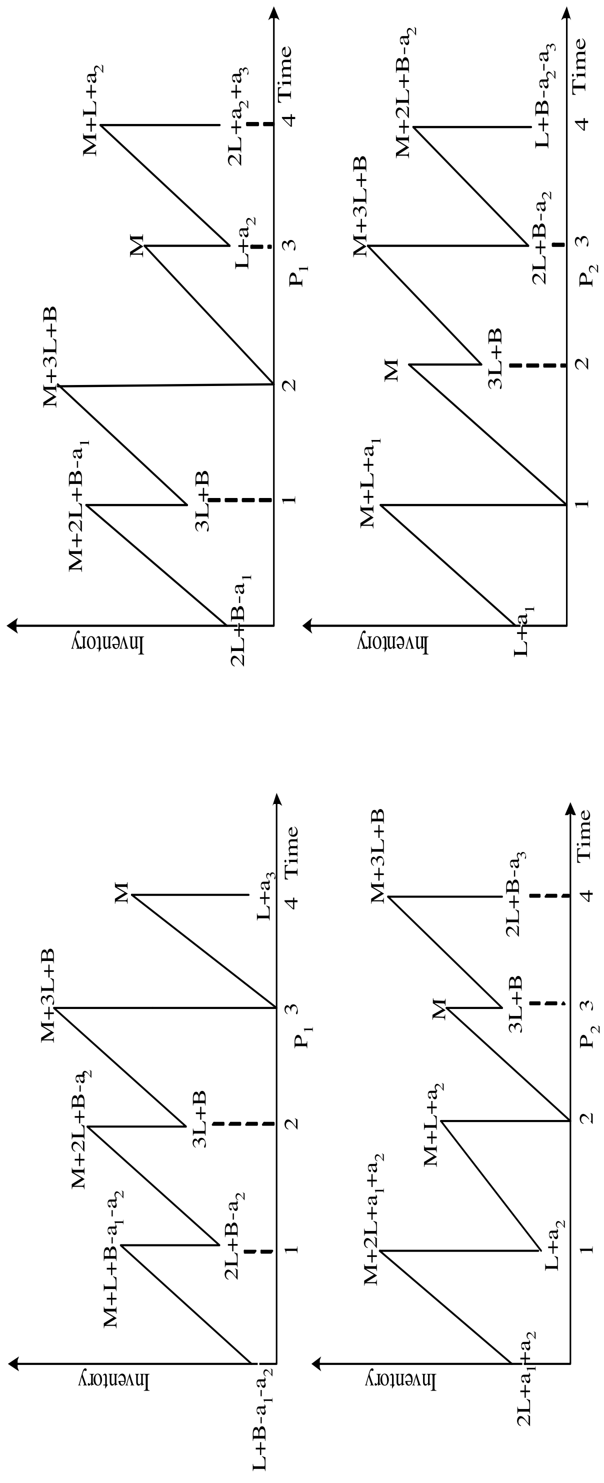
In this paper we have studied a two stage supply chain consisting of a manufacturer, a distributor, and several retailers. The JIT manufacturer produces two products that are in turn transported to the retailers (customers) by the distributor. The individual optimization objectives of the two partners (the manufacturer and the distributor) are the minimization of



(a) $\nu_p(1) = (R_{a_1}, R_{a_2}, R_{a_3}, R_{a_4})$, $I'(\nu_p(1)) = 16L + 4a_1 + 4a_2 + 4a_3$

(b) $\nu_p(1) = (R_{a_1}, R_{a_2}, R_{a_3}, R_{b_1})$, $I'(\nu_p(1)) = 12L + 4a_1 + 4a_2 + 4a_3$

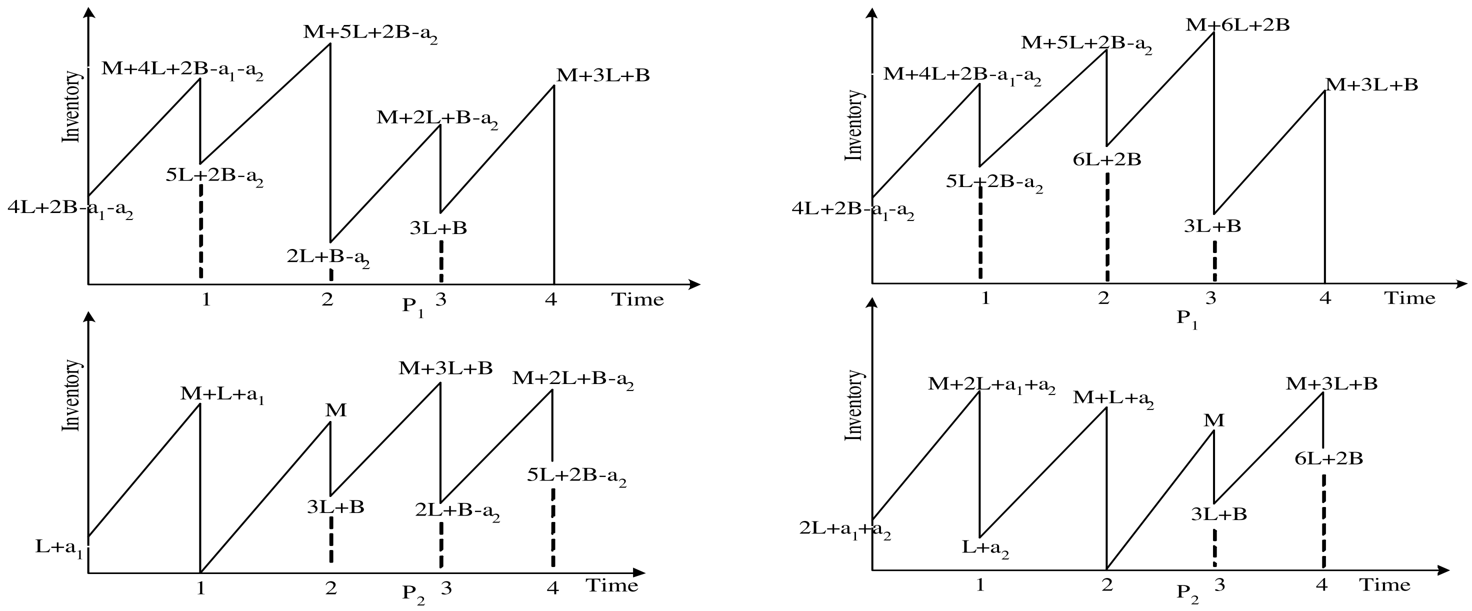
Fig. 7 Inventory graph for partial sequences $\nu_p(1) = (R_{a_1}, R_{a_2}, R_{a_3}, R_{a_4})$ and $\nu_p(1) = (R_{a_1}, R_{a_2}, R_{a_3}, R_{b_1})$



(a) $\nu_p(1) = (R_{a_1}, R_{a_2}, R_{b_1}, R_{a_3}), I'(\nu_p(1)) = 12L + 4B$

(b) $\nu_p(1) = (R_{a_1}, R_{b_1}, R_{a_2}, R_{a_3}), I'(\nu_p(1)) = 12L + 4B$

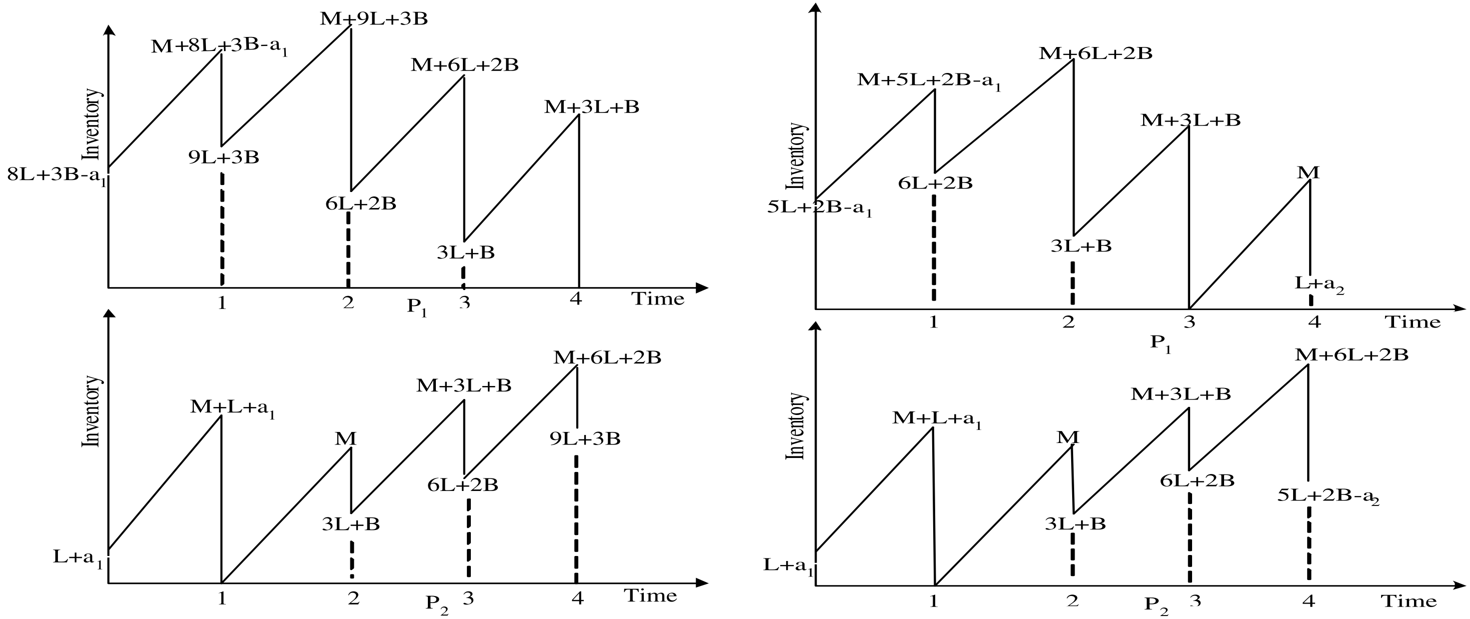
Fig. 8 Inventory graph for partial sequences $\nu_p(1) = (R_{a_1}, R_{a_2}, R_{b_1}, R_{a_3})$ and $\nu_p(1) = (R_{a_1}, R_{b_1}, R_{a_2}, R_{a_3})$



(a) $\nu_p(1) = (R_{a_1}, R_{b_1}, R_{a_2}, R_{b_2}), I'(\nu_p(1)) = 20L + 8B - 4a_2$

(b) $\nu_p(1) = (R_{a_1}, R_{a_2}, R_{b_1}, R_{b_2}), I'(\nu_p(1)) = 24L + 8B$

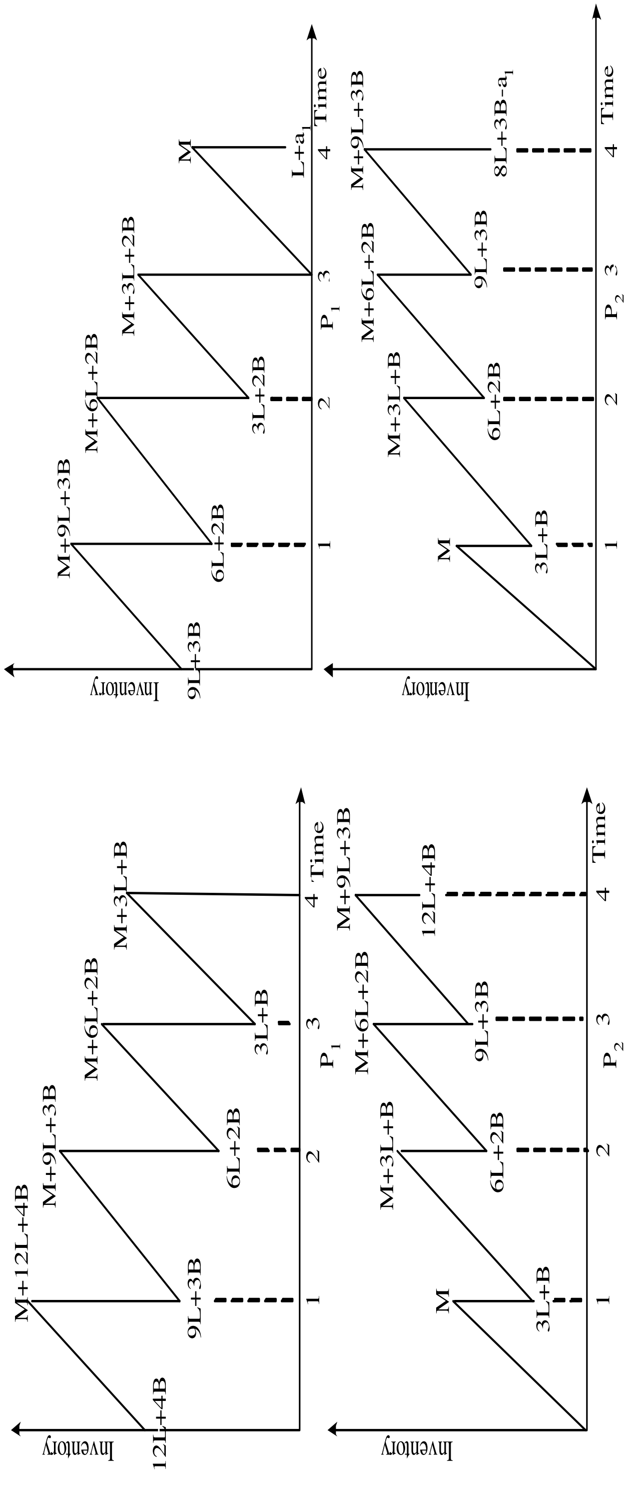
Fig. 9 Inventory graph for partial sequences $\nu_p(1) = (R_{a_1}, R_{b_1}, R_{a_2}, R_{b_2})$ and $\nu_p(1) = (R_{a_1}, R_{a_2}, R_{b_1}, R_{b_2})$



(a) $\nu_p(1) = (R_{a_1}, R_{b_1}, R_{b_2}, R_{b_3}), I'(\nu_p) = 36L + 12B$

(b) $\nu_p(1) = (R_{a_1}, R_{b_1}, R_{b_2}, R_{a_2}), I'(\nu_p) = 24L + 8B$

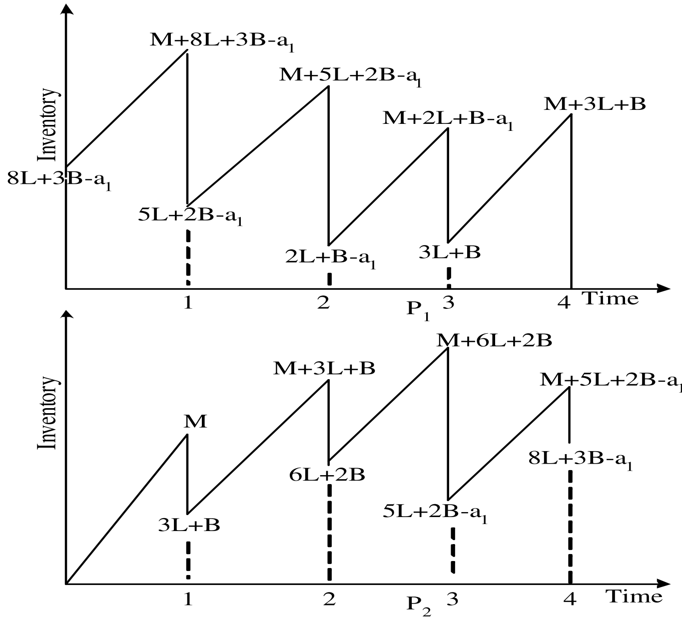
Fig. 10 Inventory graph for partial sequences $\nu_p(1) = (R_{a_1}, R_{b_1}, R_{b_2}, R_{b_3})$ and $\nu_p(1) = (R_{a_1}, R_{b_1}, R_{b_2}, R_{a_2})$



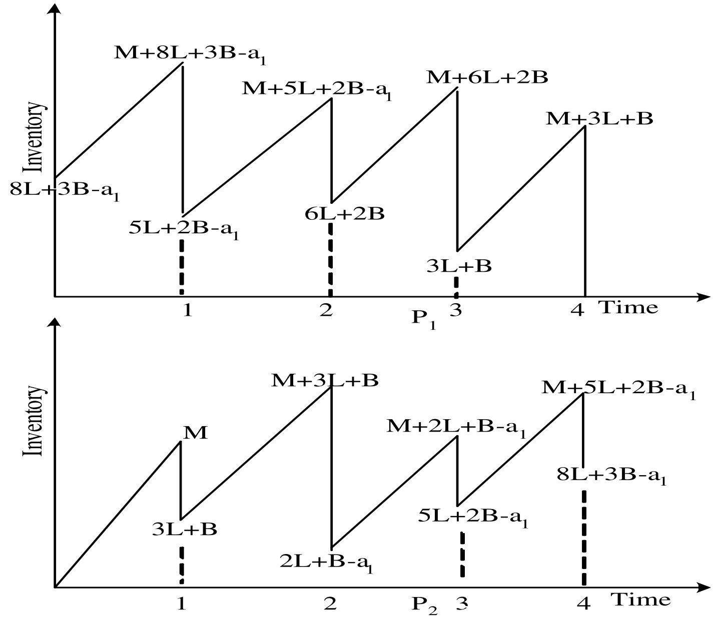
(a) $\nu_p(1) = (R_{b_1}, R_{b_2}, R_{b_3}, R_{b_4}), I'(\nu_p(1)) = 48L + 16B$

(b) $\nu_p(1) = (R_{b_1}, R_{b_2}, R_{b_3}, R_{a_1}), I'(\nu_p(1)) = 36L + 13B$

Fig. 11 Inventory graph for partial sequences $\nu_p(1) = (R_{b_1}, R_{b_2}, R_{b_3}, R_{b_4})$ and $\nu_p(1) = (R_{b_1}, R_{b_2}, R_{b_3}, R_{a_1})$

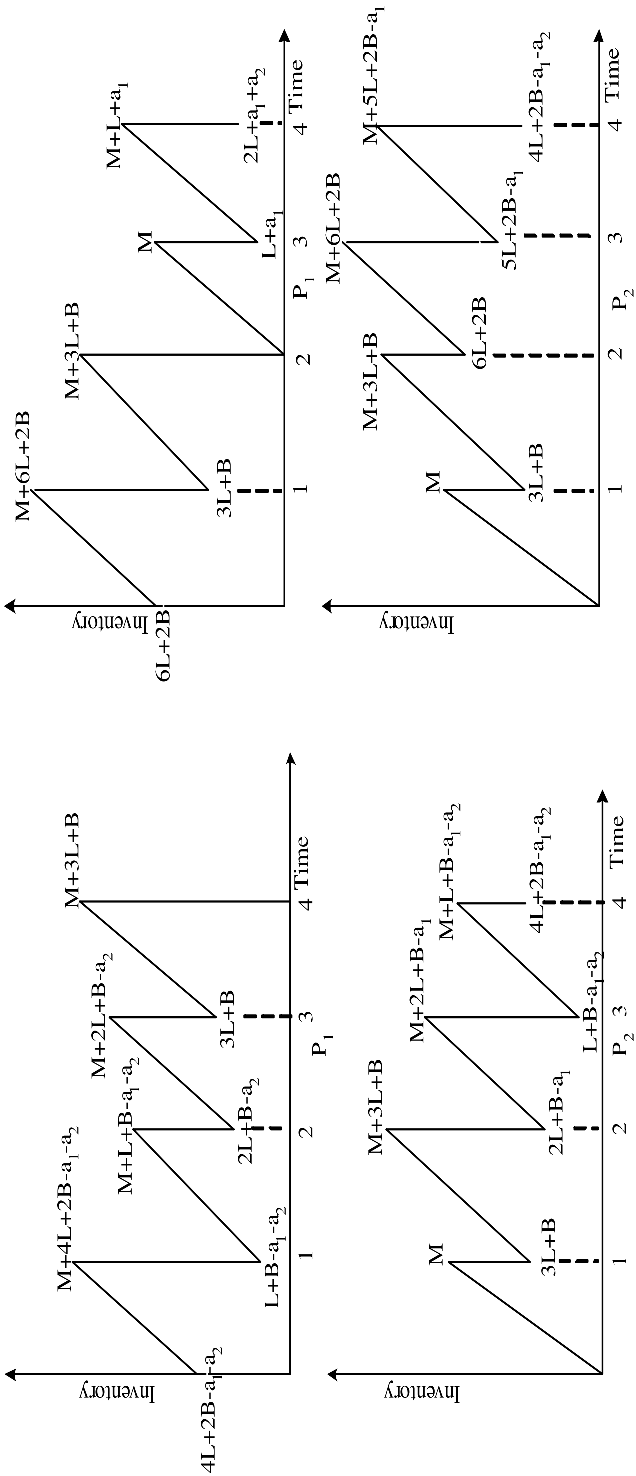


(a) $\nu_p(1) = (R_{b_1}, R_{b_2}, R_{a_1}, R_{b_3}), I'(\nu_p(1)) = 32L + 12B - 4a_1$



(b) $\nu_p(1) = (R_{b_1}, R_{a_1}, R_{b_2}, R_{b_3}), I'(\nu_p(1)) = 32L + 12B - 4a_1$

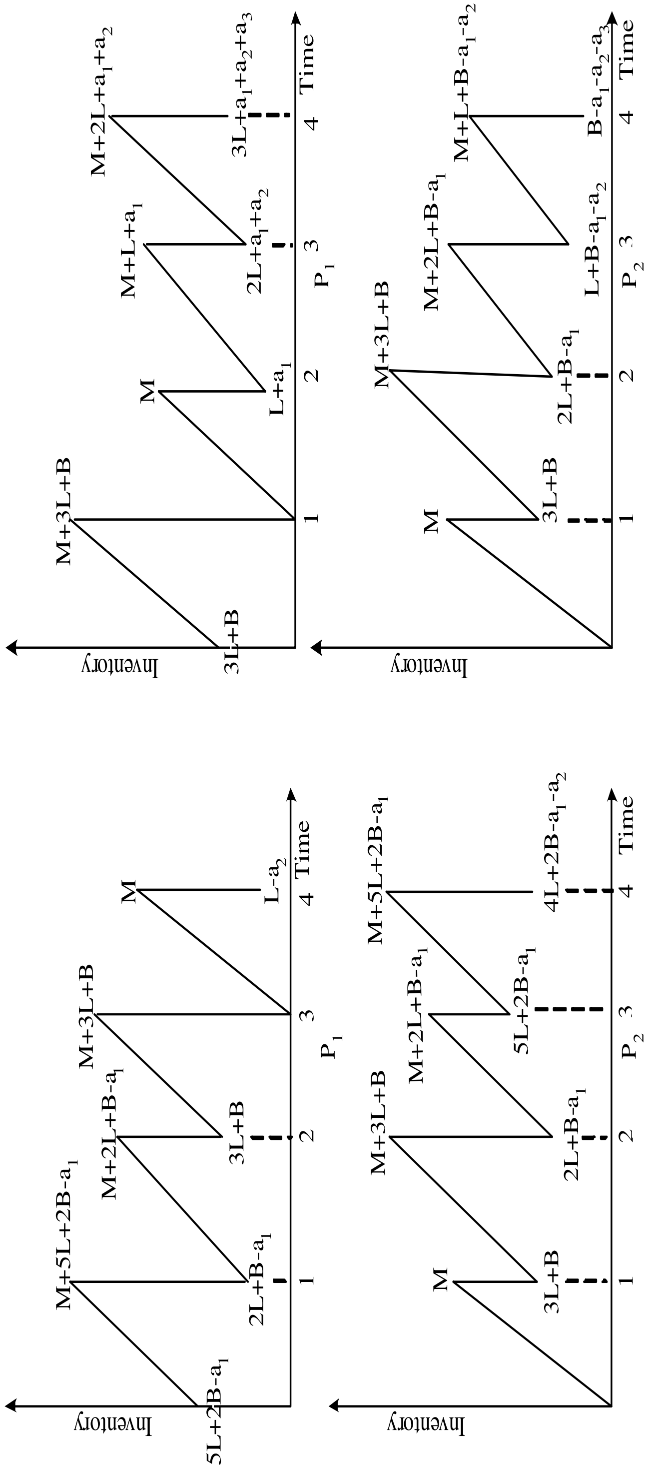
Fig. 12 Inventory graph for partial sequences $\nu_p(1) = (R_{b_1}, R_{b_2}, R_{a_1}, R_{b_3})$ and $\nu_p(1) = (R_{b_1}, R_{a_1}, R_{b_2}, R_{b_3})$



(a) $\nu_p(1) = (R_{b_1}, R_{a_1}, R_{a_2}, R_{b_2})$, $I'(\nu_p(1)) = 16L + 8B - 4a_1 - 4a_2$

(b) $\nu_p(1) = (R_{b_1}, R_{b_2}, R_{a_1}, R_{a_2})$, $I'(\nu_p(1)) = 24L + 8B$

Fig. 13 Inventory graph for partial sequences $\nu_p(1) = (R_{b_1}, R_{a_1}, R_{a_2}, R_{b_2})$ and $\nu_p(1) = (R_{b_1}, R_{b_2}, R_{a_1}, R_{a_2})$



(a) $\nu_p(1) = (R_{b_1}, R_{a_1}, R_{b_2}, R_{a_2})$, $I'(\nu_p(1)) = 20L + 8B - 4a_1 - 2a_2$

(b) $\nu_p(1) = (R_{b_1}, R_{a_1}, R_{a_2}, R_{a_3})$, $I'(\nu_p(1)) = 12L + 4B$

Fig. 14 Inventory graph for partial sequences $\nu_p(1) = (R_{b_1}, R_{a_1}, R_{b_2}, R_{a_2})$ and $\nu_p(1) = (R_{b_1}, R_{a_1}, R_{a_2}, R_{a_3})$

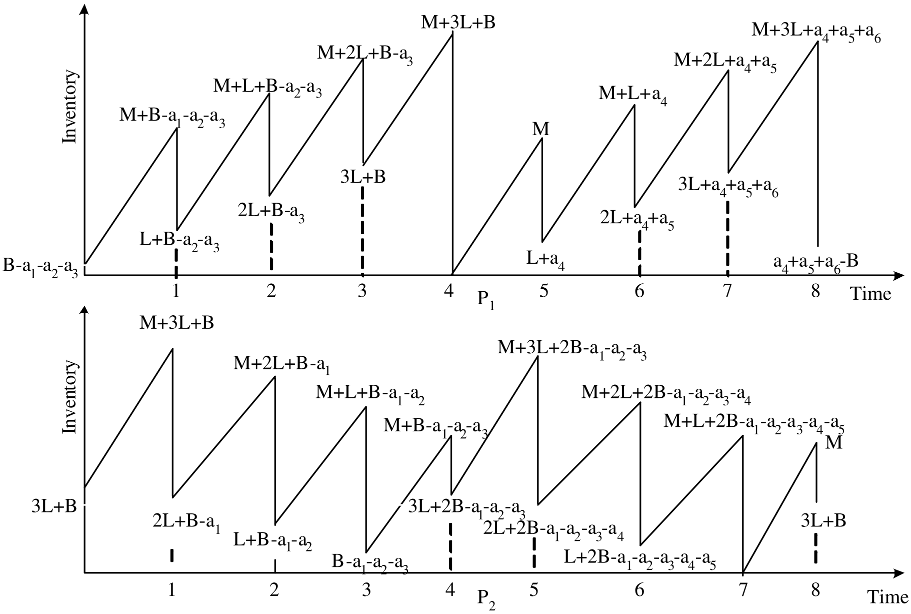


Fig. 15 $\sum_{i=1}^{3} a_i < B$, and $\sum_{i=4}^{6} a_i > B$; $\sum_{i=1}^{6} a_i = 2B$

the cost of production rate change for the manufacturer and the minimization of the inventory holding cost for the distributor. However, the production schedule and the distribution schedule, if developed independently, may produce a sub-optimal solution at the system level. In this paper we study the results of individual optimization and compare them with the results obtained for a joint optimal solution at the system level. Substantial cost savings could be achieved at the system level by joint optimization. We provide a polynomial time algorithm to solve the manufacturer's problem in which the distributor dominates. We provide an integer programming formulation for the distributor's problem where the manufacturer dominates, and also prove that the problem is strongly NP-hard. Finally, we consider the system problem that minimizes the sum of the manufacturer's cost and the distributor's cost; provide an integer programming formulation, and prove that the system problem is also strongly NP-hard.

We also define and calculate the cost of conflict for the dominated partner. Either of the two partners could be the dominated partner. We develop models to calculate the cost of conflict. Experimental results show that the cost of conflict could be significant and cooperation can reduce the total supply chain cost. We also show that joint optimization will lead to a positive surplus in the system that can be shared by the two partners to make the coordinated mechanism more attractive.

Our experiments, which used a CPLEX 8.01 linear optimizer, were limited to a maximum of 30 retailers for distributor's problem and 9 retailers for the joint optimization problem because of computational effort. Solution to large size problems will require efficient heuristics because these problems belong to the class of NP-hard problems.

Appendix

See Figs. 7–15.

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