



Short Communication

A note on the effects of downstream efficiency on upstream pricing

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ABSTRACT

The study of the effects of downstream entry on upstream pricing has revealed the counter-intuitive result of the supplier's pricing policy being invariant to a new downstream entry under an isoelastic inverse demand function. We show that this counter-intuitive result is reversed when a new downstream entry affects downstream efficiency. We show that in the presence of increased post-entry downstream efficiency the supplier increases the wholesale price by taking advantage of the increased retail efficiency and competition. We also investigate the applicability of our results under other types of inverse demand functions.

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1. Introduction

The effects of downstream entry on upstream pricing have been studied by Tyagi (1996, 1999) who concluded that in various market settings (including a Cournot oligopoly setting) and under a constant elasticity of slope inverse demand function the supplier's optimal pricing policy is invariant with respect to a new downstream entry. In Tyagi's models, the effects of entry on downstream operational efficiency are ignored by assuming the same constant pre-entry and post-entry retailer variable cost.

The purpose of this note is to show that this counter-intuitive invariance result is reversed when the effects of entry on downstream operational efficiency are incorporated. This is justified because the improved operational practices of the new entrant are usually adoptable by the current players resulting in lower post-entry variable costs for all retailers. We will show that this effect of entry on retailer operational efficiency in turn influences the supplier's pricing policy by allowing the supplier to charge a higher wholesale price to the more efficient post-entry retailers.

Tyagi (1996, 1999) and Geroski (1995) observed that the literature on entry has generally not considered the effects of entry in a given industry on the firms in a vertically related industry. Instead, typical issues studied are related to the post-entry strategies for both incumbent and entrant firms in terms of pricing, advertising, positioning, etc. (Bowman and Gatignon, 1996; Shankar, 1997; Narasimhan and Zhang, 2000).

Both Greenhut and Ohta (1976) and Tyagi (1996, 1999) have shown that the upstream input price (supplier wholesale price) does not depend on the number of downstream firms (retailers) for the constant elasticity of slope demand function and under an

entry-independent retailer variable cost. Corbett and Karmarkar (2001) extended Tyagi's model by introducing competition upstream (multiple suppliers) but assumed a linear demand function. In that case, Tyagi's price invariance result holds as well.

Mukherjee et al. (2004) considered a model with a single firm in the upstream market and a duopoly in the downstream market with one incumbent firm and one entrant. They showed that entry in the final goods market reduces input price and increases optimal profit of the incumbent if the technology of the entrant is sufficiently inferior to that of the incumbent under both quantity and price competition.

The rest of this note is organized as follows. The model formulation is presented in the next section. Our main wholesale pricing result is presented in Section 3. Extensions of our model with alternative inverse demand functions are discussed in Section 4 and the conclusions of this research are presented in Section 5.

2. The model

We consider competition in a two-tier serial supply chain comprised of n downstream retailers (at Tier 1) and one monopolist upstream supplier (at Tier 2) with the total quantity (output) of the retailers, Q , being equal to the total output of the supplier. Let the demand for the (single) final product be characterized by a general nonlinear inverse demand function $p(Q)$, where p is the retail price for the finished goods. We assume that $p(Q)$ is a thrice continuously differentiable function and that its first derivative $p' = \frac{dp}{dQ} < 0$ (we use similar notation to denote the second and third derivatives of $p(Q)$ as p'' and p''' , respectively). As in Tyagi (1996, 1999), we assume that the supplier uses a linear pricing scheme and acts as Stackelberg leader to the retailers who then compete according to the Cournot (quantity) competition framework.

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Let w be the supplier wholesale price charged to a retailer according to a linear pricing policy. The gross profit of a downstream retailer firm j is

$$\Pi_{Rj} = [p(Q) - v_j(n) - w]q_j = [p(Q_{-j} + q_j) - v_j(n) - w]q_j, \quad (1)$$

where $v_j(n)$ is the j th retailer's variable unit cost, q_j is the quantity selected by retailer j , and $Q_{-j} = Q - q_j$ is the total quantity (output) by all other retailers at Tier 1, which retailer j takes as given.

From now on we assume that Tier 1 has n identical firms, all with the same variable cost $v_j(n) = v(n)$, where $v(n)$ is a once continuously differentiable strictly decreasing function of n with its first derivative $v'(n) < 0$. The assumption that $v(n)$ is strictly decreasing quantifies the positive effects of entry on downstream operational efficiency and differentiates our model from Tyagi's (1996, 1999) models in which the retailer variable cost v is assumed entry-independent and constant. The increased post-entry retail efficiency assumption is justified by the observation that in numerous markets entry is attempted by a more efficient player utilizing his/her increased efficiency as a market penetrating tool. Subsequently, the remaining players streamline their operations to remain competitive, resulting in increased post-entry downstream efficiency for all players. For example, a new entrant may be outsourcing of labor intensive processes and/or the procurement of raw materials/components from more competitive sources compared to the current players. In response to the new entry, the pre-entry players attempt to emulate the operational efficiency of the new entrant by using similar outsourcing/procurement practices resulting in lower variable costs for all players.

The first order necessary and second order sufficient conditions for symmetric equilibria $q_j = q$ which maximize the profit in (1) can be written as

$$p'Q + np - nv(n) - nw = 0, \quad q = \frac{Q}{n} \quad (2)$$

and as $p''q + 2p' < 0$, respectively. The gross profit of the supplier, Π_S , can be written as $\Pi_S = wQ$, where we have assumed for simplicity that the variable cost of the supplier is zero. The first order necessary and second order sufficient conditions for the price w which maximizes Π_S can be written as

$$w \frac{\partial Q}{\partial w} + Q = 0 \quad (3)$$

and as $w \frac{\partial^2 Q}{\partial w^2} + 2 \frac{\partial Q}{\partial w} < 0$, respectively.

In the next section, we derive comparative statics expressions for the equilibrium value of the wholesale price w as a function of the number of retailer entrants n . All proofs are relegated to the Appendix.

3. The effects of downstream entry on the supplier's wholesale price

Proposition 1.

$$\frac{dw}{dn} = \frac{Q^2}{n^2} \frac{[-p'p'' + Q(p'')^2 - Qp'p''']}{2(n+1)p' + (n+4)Qp'' + Q^2p'''} - v'(n) \frac{[(n+1)p' + (n+3)Qp'' + Q^2p''']}{2(n+1)p' + (n+4)Qp'' + Q^2p'''} \quad (4)$$

In general, the derivative in Proposition 1 cannot be signed for general inverse demand functions $p(Q)$. Proposition 1 can be compared with the corresponding results of Tyagi (1999) (derived under the assumption of an entry-independent constant retailer variable cost, $v(n) = v$). The second term in (4) vanishes in Tyagi's (1999) model because $v'(n) = 0$ when $v(n)$ is assumed to be constant, independent of n . Furthermore, if we assume an isoelastic inverse demand

function as in Tyagi (1999) of the form $p(Q) = a - bQ^\beta$, with $a > 0, b > 0$, and $\beta > 0$, the numerator of the first term in (4) becomes equal to zero yielding the counter-intuitive result of the wholesale price w being invariant in the number of retailers n . However, when $p(Q) = a - bQ^\beta$ is substituted in (4), we obtain $\frac{dw}{dn} = -\frac{\beta}{1+\beta} v'(n) > 0$, that is, the wholesale price w increases with the number of retailers n .

This finding reverses the counter-intuitive wholesale price invariance result of Tyagi (1999). In our model, the supplier takes advantage of the increased post-entry competition among retailers and raises the wholesale price. An alternative interpretation of the supplier's action is that the supplier recognizes the increased profit potential of the post-entry retailers due to their increased efficiency and responds by increasing the wholesale price.

We demonstrate our findings in the following numerical example. Assume that $p(Q) = 1 - Q$, that is $a = b = \beta = 1$ and that $v(n) = \frac{1}{n}$. In that case, (4) reduces to $\frac{dw}{dn} = \frac{1}{2n^2}$ which in turn yields $w = c - \frac{1}{2n}$ for some constant c . The last expression clearly indicates that the wholesale price w increases as the number of retailers increases because of a new entry.

4. Extensions

Our model's limitation is its dependence on a specific pricing scheme (linear pricing) and on a specific type of inverse demand function (isoelastic). An alternative pricing scheme like two-part pricing in which the manufacturer also collects a fixed fee from each retailer cannot be used in our model because it changes the competition structure at the retailer level as each retailer earns zero profit in a two-part pricing scheme.

With respect to the consideration of inverse demand functions other than the isoelastic one, Tyagi (1999) proves in his Corollary 3.1 a necessary and sufficient condition for his wholesale price invariance result to hold (in the absence of increased post-entry downstream efficiency). Tyagi's (1999) Corollary 3.1 states that the only inverse demand functions leading to this invariance result are of an isoelastic form. Since the derivatives in (4) cannot be signed for a general inverse demand function, it is of interest to identify the type and/or the properties of other potential inverse demand functions which can be used in place of the isoelastic inverse demand function in our model and yield similar results. A careful examination of (4) reveals that if $p' < 0, p'' < 0, p''' < 0$ and $(p'')^2 - p'p''' = c \leq 0$ (where c is a constant), then $\frac{dw}{dn} > 0$ for all $Q > 0$ (because $v'(n) < 0$).

Proposition 2. *The conditions $p(0) > 0, p' < 0, p'' < 0, p''' < 0$ and $(p'')^2 - p'p''' = c \leq 0$ (where c is a constant) hold if and only if $p(Q) = a - be^{kQ}$, where $a > b > 0, k > 0$ and $c = 0$.*

Proposition 2 enables us to identify the class of functions $p(Q) = a - be^{kQ}$, where $a > b > 0$ and $k > 0$, as the unique class of functions for which $p(0) > 0, p' < 0, p'' < 0, p''' < 0, (p'')^2 - p'p''' = c \leq 0$ and therefore $\frac{dw}{dn} > 0$ for all $Q > 0$.

5. Conclusions

We incorporated the effects of entry on downstream operational efficiency by modeling the retailer's variable cost as a decreasing function of the number of retailers. Our model showed that the supplier responds to this improved retailer efficiency by increasing the post-entry wholesale price. We showed that in the presence of increased post-entry downstream efficiency the supplier increases the wholesale price by taking advantage of the increased retail efficiency and competition. We also investigated the applicability of our results under other types of inverse demand functions.

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Appendix

Lemma 1.

$$\frac{dQ}{dn} = \frac{\frac{Q}{n}p'}{(n+1)p' + Qp''} + n \frac{1}{(n+1)p' + Qp''} \left[\frac{dw}{dn} + v'(n) \right]. \tag{5}$$

Proof. Observe that $w = w(n)$, $Q = Q(n, w(n))$ and recall that the first order condition for Q which maximizes the profit in (1) is given by (2). Then, by taking the derivative of the (2) we obtain

$$\begin{aligned} \frac{d}{dn} [p'Q + np - nv(n) - nw] &= \frac{dQ}{dn} p' + Qp'' \frac{dQ}{dn} + p + np' \frac{dQ}{dn} - v(n) \\ &\quad - nv'(n) - w - n \frac{dw}{dn} \\ &= 0. \end{aligned}$$

This yields

$$\frac{dQ}{dn} = \frac{-p + w + v(n)}{(n+1)p' + Qp''} + n \frac{1}{(n+1)p' + Qp''} \left[\frac{dw}{dn} + nv'(n) \right]. \tag{6}$$

Utilizing (2) we obtain $-p + v(n) + w = p' \frac{Q}{n}$, which, together with (6) yields (5). □

Proof of Proposition 1. We rewrite (3) as

$$w = -\frac{Q}{\frac{\partial Q}{\partial w}}. \tag{7}$$

By taking the partial derivative of (2) we obtain

$$\frac{\partial}{\partial w} [p'Q + np - nv(n) - nw] = \frac{\partial Q}{\partial w} p' + Qp'' \frac{\partial Q}{\partial w} + np' \frac{\partial Q}{\partial w} - n = 0,$$

which yields

$$\frac{\partial Q}{\partial w} = \frac{n}{(n+1)p' + Qp''}. \tag{8}$$

The combination of (7) and (8) yields

$$w = -\frac{Q}{n} [(n+1)p' + Qp''] = -Q \left(1 + \frac{1}{n} \right) p' - Q^2 \left(\frac{1}{n} \right) p'',$$

and

$$\frac{dw}{dn} = -\frac{1}{n} \frac{dQ}{dn} [(n+1)p' + (n+3)Qp'' + Q^2 p'''] + \frac{Qp' + Q^2 p''}{n^2}. \tag{9}$$

Define $A = (n+1)p' + Qp''$ and $B = (n+1)p' + (n+3)Qp'' + Q^2 p'''$. Expressions (5) and (9) can be written as

$$\frac{dQ}{dn} = \frac{1}{A} \left[\frac{Q}{n} p' + nv'(n) + n \frac{dw}{dn} \right], \tag{10}$$

and

$$\frac{dw}{dn} = -\frac{B}{n} \frac{dQ}{dn} + \frac{Qp' + Q^2 p''}{n^2}, \tag{11}$$

respectively. Expression (11) implies that

$$\frac{dQ}{dn} = -\frac{n}{B} \frac{dw}{dn} + \frac{Qp' + Q^2 p''}{nB}. \tag{12}$$

By equating (10) and (12) we obtain after some simplifications that

$$\frac{dw}{dn} = \frac{A(Qp' + Q^2 p'') - BQp'}{n^2(A+B)} - \frac{v_1'(n)B}{A+B}. \tag{13}$$

Since $A+B = 2(n+1)p' + (n+4)Qp'' + Q^2 p'''$ and $A(Qp' + Q^2 p'') - BQp' = Q^2[-p'p'' + Q(p'')^2 - Qp'p''']$, we obtain (4) from (13). □

Proof of Proposition 2. It is easy to show that $p(0) > 0, p' < 0, p'' < 0, p''' < 0$ and $(p'')^2 - p'p''' = 0$ when $p(Q) = a - be^{kQ}$, where $a > b > 0$ and $k > 0$.

Let $p'(\cdot) \equiv g(\cdot)$. The differential equation $(p'')^2 - p'p''' = c$ with $c < 0$ is equivalent to $(g')^2 - gg'' = c$. This second order differential equation admits two closed form solutions for g :

$$g(x) = -\frac{1}{2} e^{-xe^{c_1} - 2c_1 - c_2 e^{c_1}} (ce^{2c_1} - e^{2(x+c_2)e^{c_1}}) \tag{14}$$

or

$$g(x) = -\frac{1}{2} e^{-xe^{c_1} - 2c_1 - c_2 e^{c_1}} (ce^{2xe^{c_1} + 2c_1 + 2c_2 e^{c_1}} - 1), \tag{15}$$

where $c_i, i = 1, 2$ are constants.

Given that $c < 0$, it is easy to see that both (14) and (15) yield $g(x) > 0$ and hence $p'(\cdot) > 0$. Therefore, there exist no functions that satisfy $p' < 0, p'' < 0, p''' < 0$ and $(p'')^2 - p'p''' < 0$. For $c = 0$, the solution to $(g')^2 - gg'' = 0$ is $g(x) = c_2 e^{xc_1}$ and $p(x) = \int g(x)dx = c_3 + \frac{c_2}{c_1} e^{xc_1}$. Defining $b = -\frac{c_2}{c_1}, k = c_1, a = c_3 > b$, we obtain $p(x) = a - be^{kx}$. Hence, $c_2 < 0$ guarantees that $g < 0$ ($p' < 0$), and $c_1 > 0$ guarantees that $g' < 0$ ($p'' < 0$) and $g'' < 0$ ($p''' < 0$). The condition $a = c_3 > b$ guarantees that $p(0) > 0$. The class of functions of the form $p(x) = c_3 + \frac{c_2}{c_1} e^{xc_1}$ is the only solution to $p' < 0, p'' < 0, p''' < 0$ and $(p'')^2 - p'p''' \leq 0$. □

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