



Short Communication

A note on the classification of consumer demand functions with respect to retailer pass-through rates

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ABSTRACT

Tyagi (1999) derived conditions on the curvature of consumer demand functions which make it optimal for a profit-maximizing retailer to pass-through greater (less) than 100% of a manufacturer trade deal amount. Since the pass-through is customarily evaluated at the optimal wholesale price, then additional sufficient conditions are needed to ensure the existence of an optimal wholesale price. The purpose of this note is to derive the additional required conditions on the curvature of the consumer demand functions for the existence of a greater (less) than 100% retailer pass-through rate at the optimal wholesale price.

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1. Introduction

Tyagi (1999) derived conditions on the curvature of consumer demand functions which make it optimal for a profit-maximizing retailer to pass-through greater (less) than 100% of a manufacturer trade deal amount. Since the pass-through is customarily evaluated at the optimal wholesale price, then additional sufficient conditions are needed to ensure the existence of an optimal wholesale price. In this note, we derive the additional required conditions on the curvature of the consumer demand functions for the existence of a greater (less) than 100% retailer pass-through rate at the optimal wholesale price.

2. The model

Tyagi (1999) considered a model in which a manufacturer sells a product to a retailer using a linear pricing policy. The retailer, in turn, sells the product to the consumer. The manufacturer sets the wholesale price w acting as a Stackelberg price leader and the retailer sets the retail price p acting as a Stackelberg price follower. The consumer demand function is $q(p)$. Denote the first order, second order and third order derivatives of $q(p)$ w.r.t. p as $q'(p)$, $q''(p)$ and $q'''(p)$, respectively. Tyagi (1999) assumed that the following condition holds:

(A0) $q(p)$ is thrice continuously differentiable and $q'(p) < 0$ for all $p > 0$.

The retailer's profit-maximization problem is

$$\max_{p \geq 0} \Pi_R(p) = (p - w)q(p). \quad (1)$$

If this problem has an optimal solution $p(w)$ for any $w \geq 0$, then, the optimal retail price function corresponding to w is $p(w)$. The manufacturer's profit-maximization problem is

$$\max_{w \geq 0} \Pi_M(w) = wq[p(w)]. \quad (2)$$

If this problem has an optimal solution $w^* \geq 0$ then, the resulting optimal retail price is $p^* = p(w^*) \geq 0$. The retailer pass-through at the optimal wholesale price is defined as $\frac{dp}{dw}(w^*)$, that is, as the derivative of the optimal retail price function $p(w)$ evaluated at the optimal wholesale price w^* . It should be pointed out that under Tyagi's (1999) assumption (A0), either one of the above two maximization problems may not, in general, have a solution which means that the retailer pass-through $\frac{dp}{dw}(w^*)$ cannot be evaluated. To illustrate this possibility, consider Example 3 in Tyagi (1999, p. 154), in which $q(p) = p^\beta$, $\beta < -1$. For ease of presentation assume that $\beta = -2$. Then $q(p) = p^{-2}$ is thrice continuously differentiable and $q'(p) = -2p^{-3} < 0$ for $p > 0$, therefore, assumption (A0) holds. The retailer's profit-maximization problem (1) becomes

$$\max_{p \geq 0} \Pi_R(p) = (p - w)p^{-2}. \quad (3)$$

For any $w > 0$, the unique optimum in (3) is $p(w) = 2w$ and $\Pi'_R(p) = 0$, $\Pi''_R(p) < 0$ at $p(w) = 2w$. The manufacturer's profit-maximization problem (2) becomes

$$\max_{w \geq 0} \Pi_M(w) = wq(2w) = \frac{1}{4w}. \quad (4)$$

Problem (4) has no optimal solution since $\Pi_M(w) \rightarrow \infty$ as $w \rightarrow 0^+$. Therefore, the optimal prices w^* and $p(w^*)$ do not exist and the pass-through $\frac{dp}{dw}(w^*)$ cannot be evaluated at w^* for this problem.

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Since $\frac{dp}{dw}$ is customarily evaluated at w^* , the above observations necessitate the derivation of additional conditions on the curvature of $q(p)$ which will ensure the existence of w^* and $\frac{dp}{dw}(w^*)$.

Theorem 1. Suppose that the following assumptions hold in addition to assumption (A0).

- (A1) $0 < q(p) < \infty$ for all $p \geq 0$.
- (A2) $p q(p) \rightarrow 0$ as $p \rightarrow \infty$.
- (A3) $f(p) = \ln q(p)$ satisfies $f'(p) + p f''(p) < 0$ for all $p > 0$.

Then, the following conclusions hold.

- (1) The retailer's profit-maximization problem (1) has a unique optimal solution $p(w) > w$ for all $w \geq 0$.
- (2) The manufacturer's profit-maximization problem (2) has an optimal (possibly non-unique) solution $w^* > 0$.
- (3) The optimal price function $p(w)$ is twice continuously differentiable for $w > 0$ and the retailer pass-through at w^* is given by

$$\frac{dp}{dw}(w^*) = \frac{[q'(p^*)]^2}{2[q'(p^*)]^2 - q(p^*)q''(p^*)}, \tag{5}$$

where $p^* = p(w^*) > 0$.

Assumption (A1) requires the consumer demand function $q(p)$ to be positive and finite for all nonnegative prices p . The economic justification of the fact that the consumer demand function is bounded can be attributed to the finite wealth of the economy. Assumption (A2) requires the total retailer revenue function $p q(p)$ to decline to zero as the retail price p becomes very large. This behavior can be attributed to the fact that customers are somewhat rational so the number of buyers decreases faster than the increase in price making the transaction less profitable to the retailer. Assumption (A3) requires the slope of the natural logarithm of the demand function not to increase too fast. In general, when $\ln q(p)$ is concave (convex), the $q(p)$ function is called log-concave (log-convex). The marketing interpretation of log-concavity is that a price increase reduces the quantity more at high prices than it does at low prices.

Consider again Example 3 in Tyagi (1999, p. 514) in which $q(p) = p^\beta$, $\beta < -1$. Since $q(0) = \infty$, assumption (A1) does not hold. Also, since $f(p) = \ln q(p) = \beta \ln p$, $f'(p) + p f''(p) = 0$ for $p > 0$, therefore assumption (A3) does not hold as well. On the other hand, assumption (A2) holds since $p q(p) = p^{1+\beta} \rightarrow 0$ as $p \rightarrow \infty$. The failure of assumptions (A1) and (A3) to hold results in the non-existence of the optimal prices w^* and $p(w^*)$ for $\beta = -2$ (even though $p(w)$ is well defined for $w > 0$). It is easy to construct an example in which assumption (A2) does not hold and, as a result, problem (1) does not have an optimal solution (e.g., when $q(p) = p^\beta$ with $-1 < \beta < 0$). On the other hand, it is easy to verify that all assumptions (A1)–(A3) hold in Examples 1 and 2 in Tyagi (1999).

Tyagi (1999) defined the construct $\phi = \frac{q(p)q''(p)}{[q'(p)]^2}$ which relates change in price to change in marginal revenue and used expression (5) (which implies that $\frac{dp}{dw} = \frac{1}{2-\phi}$) to prove the following: $\frac{dp}{dw} < 1$ if and only if $\phi < 1$, $\frac{dp}{dw} = 1$ if and only if $\phi = 1$ and $\frac{dp}{dw} > 1$ if and only if $1 < \phi \leq 2$.

Our next proposition generalizes Corollaries 1 and 2 in Tyagi (1999) so that $\frac{dp}{dw}$ can be properly evaluated at w^* . For completeness of our presentation, we state these two corollaries next.

Corollary 1 (Tyagi, 1999). The retail pass-through is always less than 100% for the linear demand function and for all concave demand functions.

Corollary 2 (Tyagi, 1999). The retail pass-through can be greater than 100% for convex demand functions.

It should be pointed out that Amir (2005) derived results analogous to Proposition 1 for monopoly pass-through using the theory of supermodular optimization in which $p(w)$ is not assumed to be unique and the derivatives of $p(w)$ are not utilized.

Proposition 1. Suppose that assumption (A0) holds and that $\frac{dp}{dw}$ is given by (5). Then,

- 1-1. $\frac{dp}{dw} \leq 1$ if and only if $f(p) = \ln q(p)$ is concave.
- 1-2. $\frac{dp}{dw} \geq 1$ if and only if $f(p) = \ln q(p)$ is convex and $f'(p) \leq [f'(p)]^2$.

When $f(p) = \ln q(p)$ is concave, then $f'(p) \leq 0$ and assumption (A3) holds since $f'(p) < 0$ by (A0). In this case, $\frac{dp}{dw} \leq 1$ (by Proposition 1) which generalizes Corollary 1 in Tyagi (1999) since the assumption that $f(p) = \ln q(p)$ is concave is more general than the assumption that $q(p)$ is either linear or concave. On the other hand, when $f(p) = \ln q(p)$ is convex, then $f'(p) \geq 0$ and assumption (A3) may not hold. In this case, $\frac{dp}{dw} \geq 1$ if $f'(p) \leq [f'(p)]^2$ (by Proposition 1) which generalizes Corollary 2 in Tyagi (1999) since the assumptions that $f(p) = \ln q(p)$ is convex and $f'(p) \leq [f'(p)]^2$ apply to some convex functions.

3. Conclusions

We derived sufficient conditions on the curvature of the consumer demand function for the existence of the optimal wholesale price w^* . These conditions facilitate the evaluation of the retailer pass-through at w^* . Based on these conditions, the negative exponential demand function and the varying elasticity demand function lead to a greater than 100% retailer pass-through; in contrast, the retailer pass-through cannot be evaluated at w^* for the constant elasticity demand function due to lack of existence of w^* in that case. Future research should focus on deriving the appropriate conditions for retailer pass-through in more complex market settings.

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Appendix A

Proof of Theorem 1

Part (1). Let $w \geq 0$ be fixed and consider $\Pi_R(p) = (p - w)q(p)$. By (A1), $q(p) > 0$ for $p \geq 0$. Thus, for $p > w$, $\Pi_R(p) > 0$, for $p \leq w$, $\Pi_R(p) \leq 0$ and $\max_{p \geq 0} \Pi_R(p) = \max_{p > w} \Pi_R(p)$. Observe that, by (A0), $q(p)$ is continuous and $\Pi_R(p) \rightarrow 0$ as $p \rightarrow w^+$ and that, by (A2), $\Pi_R(p) \rightarrow 0$ as $p \rightarrow \infty$. Thus, for some $p_2 > p_1 > w$, $\max_{p \geq 0} \Pi_R(p) = \max_{p \in [p_1, p_2]} \Pi_R(p)$. Since, by (A0), $\Pi_R(p)$ is continuously differentiable, this implies that $\max_{p \geq 0} \Pi_R(p) = \Pi_R(p(w))$ for some $p(w) > w$ and the corresponding first order necessary condition

$$\Pi'_R(p) = q(p) + (p - w)q'(p) = 0 \tag{6}$$

holds at $p(w) > w$. By (A3), $f(p) = \ln q(p)$ which implies that $q(p) = \exp(f(p))$. Observe that by (A0)–(A1), $f'(p) = \frac{q'(p)}{q(p)} < 0$ for all $p > 0$ and condition (6) can be written equivalently as

$$\Pi'_R(p) = [1 + (p - w)f'(p)]\exp(f(p)) = 0. \tag{7}$$

Define the function $g(p, w) = 1 + (p - w)f'(p)$. In view of (7), condition (6) holds at $p(w) > w$ if and only if $g(p, w) = 0$ at $p(w) > w$. The partial derivative of $g(p, w)$ with respect to p is given by

$$\frac{\partial g}{\partial p}(p, w) = f'(p) + (p - w)f''(p). \tag{8}$$

Since $f'(p) < 0$ for all $p > 0$, condition $f(p) + p f'(p) < 0$ implies, in view of (8), that $\frac{\partial g}{\partial p}(p, w) < 0$ for all $p > w$. Thus, at $p = p(w) > w$, $g(p, w) = 0$ and $\frac{\partial g}{\partial p}(p, w) < 0$. Recall that since $w \geq 0$ was assumed fixed, these conditions hold for all $w \geq 0$. Also, by (A0), $f(p)$ is thrice continuously differentiable in p and consequently, $g(p, w)$ is twice continuously differentiable in p . Therefore, by the implicit function theorem, there exists a unique twice continuously differentiable function $p(w)$, $p(w) > w$, which satisfies $g(p, w) = 0$ for all $w > 0$. Recall that for all $w \geq 0$, $\max_{p \geq 0} \Pi_R(p) = \Pi_R(p(w))$ for some $p(w) > w$. Since the first order necessary condition (6) holds at $p(w) > w$ if and only if $g(p, w) = 0$ at $p(w) > w$, the retailer's profit-maximization problem (1) has a unique optimal solution $p(w) > w$ for all $w \geq 0$ which proves part (1).

Part (2). Consider $\Pi_M(w) = wq[p(w)]$ for $w \geq 0$. By Part (1), $p(w) > w$ exists and is unique for all $w \geq 0$. Define the function $\Pi(w) = wq(w)$ for $w \geq 0$. By (A0)–(A1) and the observation that $p(w) > w$, $q[p(w)] \leq q(w)$ and $0 \leq \Pi_M(w) \leq \Pi(w)$ for all $w \geq 0$. Since $\Pi_M(0) = 0$ and $\Pi_M(w) > 0$ for $w > 0$, $\max_{w \geq 0} \Pi_M(w) = \max_{w > 0} \Pi_M(w)$. Observe that, by (A0)–(A1), $q(w)$ is continuous and $\Pi(w) \rightarrow 0$ as $w \rightarrow 0+$ and, by (A2), $\Pi(w) \rightarrow 0$ as $w \rightarrow \infty$. Therefore, $\Pi_M(w) \rightarrow 0$ as $w \rightarrow 0+$ and $\Pi_M(w) \rightarrow 0$ as $w \rightarrow \infty$. Thus, for some $w_2 > w_1 > 0$, $\max_{w \geq 0} \Pi_M(w) = \max_{w \in [w_1, w_2]} \Pi_M(w)$. Since, by Part (1), $p(w)$ is twice continuously differentiable for $w > 0$, by (A0), $\Pi_M(w)$ is also twice continuously differentiable for $w > 0$. Therefore, since $\Pi_M(w)$ is continuous and the set $[w_1, w_2]$ is nonempty and compact, by the classical Weierstrass theorem $\max_{w \in [w_1, w_2]} \Pi_M(w) = \Pi_M(w^*)$ for some $w^* \in [w_1, w_2]$. This implies that $\max_{w \geq 0} \Pi_M(w) = \Pi_M(w^*)$ for some $w^* > 0$, that is, the manufacturer's profit-maximization problem (2) has an optimal (possibly non-unique) solution $w^* > 0$.

Part (3). By Part (1), there exists a unique twice continuously differentiable function $p(w)$, $p(w) > w$, which satisfies $g(p, w) = 1 + (p - w)f'(p) = 0$ and $\frac{\partial g}{\partial p}(p, w) = f'(p) + (p - w)f''(p) < 0$ for all $w > 0$. By the implicit function theorem,

$$\frac{dp}{dw}(w^*) = -\frac{\frac{\partial g}{\partial w}(p^*, w^*)}{\frac{\partial g}{\partial p}(p^*, w^*)} = -\frac{-f'(p^*)}{f'(p^*) + (p^* - w^*)f''(p^*)}, \tag{9}$$

where $p^* = p(w^*) > 0$. Since $g(p^*, w^*) = 1 + (p^* - w^*)f'(p^*) = 0$, $p^* - w^* = \frac{-1}{f'(p^*)}$ and (9) implies that

$$\frac{dp}{dw}(w^*) = \frac{[f'(p^*)]^2}{[f'(p^*)]^2 - f''(p^*)}, \tag{10}$$

where $p^* = p(w^*) > 0$. Since $f(p) = \ln q(p)$, $f'(p) = \frac{q'(p)}{q(p)}$ and $f''(p) = \frac{q(p)q''(p) - [q'(p)]^2}{[q(p)]^2}$. The substitution of $f(p^*)$, $f'(p^*)$ into (10) yields (5). \square

Proof of Proposition 1

Part 1-1. Let $f(p) = \ln q(p)$. By (A0), $q(p)$ and thus $f(p)$ are continuously differentiable; also, $q'(p) < 0$ for all $p > 0$. Then, $f''(p) = [\ln q(p)]'' = \frac{q(p)q''(p) - [q'(p)]^2}{[q(p)]^2}$. Thus, $f''(p) \leq 0$ if and only if $q(p)q''(p) - [q'(p)]^2 \leq 0$ which is equivalent to $\phi \leq 1$. Since $f'(p) \leq 0$ if and only if $f(p)$ is concave, the conclusion in 1-1 follows from Tyagi's (1999) conclusion that $\frac{dp}{dw} \leq 1$ if and only if $\phi \leq 1$.

Part 1-2. The proof that $\phi \geq 1$ if and only if $f(p)$ is convex is analogous to the proof of Part 1-1. Since $f'(p) = \frac{q'(p)}{q(p)}$ and $f''(p) = \frac{q(p)q''(p) - [q'(p)]^2}{[q(p)]^2}$, $f''(p) \leq [f'(p)]^2$ if and only if $q(p)q''(p) - [q'(p)]^2 \leq [q'(p)]^2$ which is equivalent to $\phi \leq 2$. Then, the conclusion in 1-2 follows from Tyagi's (1999) conclusion that $\frac{dp}{dw} \geq 1$ if and only if $1 \leq \phi \leq 2$. \square

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